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Combining Differentiation and Challenge in Mathematics Instruction: A Case from Practice

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Despite the challenges of teaching mathematics to diverse learners and the importance of students engaging in cognitively demanding tasks, practice-based research on instruction that combines these priorities is relatively rare. In this paper, a primary mathematics laboratory is used as the setting for implementing such instruction. Records of practice – including video records of teaching, lesson plans, and student work – are analysed to identify task characteristics and teacher actions that supported differentiated instruction and work on challenging tasks. Among the themes identified are the teacher actively guiding the students' learning, the establishment of classroom norms, and enabling all students to act as resources for one another's learning.

Keywords: Individualised instruction, Productive struggle, Problem sets, Difficulty level, Teaching methods, Classroom environment.

Introduction.

In many countries classrooms are increasingly diverse and teachers need to cater for students from a wide variety of intellectual, linguistic, cultural, social and economic backgrounds. However, relatively few supports are available to help teachers differentiate instruction so that all students, despite their differences, can meaningfully participate in mathematics lessons. Furthermore, using challenging tasks has the potential to improve students' learning (e.g. Stein, Grover and Henningsen, 1996) by engaging students in productive struggle (Warshauer, 2015). One problem is that sometimes in classroom practice only high-achieving students are challenged or differentiated instruction is intended only for some students. Until relatively recently, few studies combined a focus on challenging students mathematically while simultaneously differentiating instruction (e.g. Brown et al, 2017; Lynch et al, 2018; Sullivan et al, 2013). This paper presents a case study of one teacher attempting to engage students in cognitively demanding work over the course of a lesson while simultaneously differentiating instruction.

The following research question is addressed: What steps taken by the teacher supported differentiation and maintained or modified the challenge of the task?

Theoretical Framework.

Two areas of research inform this study. The first is the mathematical task framework proposed by Stein and Smith (1998) which classifies mathematical tasks into those with lower-level demands (consisting of memorisation tasks and procedures without connections) and higher-level demands (consisting of procedures with connections and doing mathematics). This framework was necessary to identify times when the teacher maintained, raised or lowered the cognitive demand of the task. Although specific tasks may be initially classified using these categories, the classification may change over the course of instruction, as the teacher sets up the task for students and as the task is implemented in the classroom.

Tomlinson's work (2000) on differentiating instruction provides the second pillar of the framework because the teacher's goal is for all students to experience challenge in line with their readiness for it. Tomlinson identifies four elements which teachers can differentiate to meet the needs of all students: content, process, product, learning environment. Content of tasks can be differentiated by presenting information to students in multiple communication modes such as written or oral, or by providing supports to help them read the text. Differentiating the process allows students to complete a task in various ways depending on their preferences; this may involve providing materials, varying the time allocated, permitting individual or group effort, or breaking a task into smaller parts for some students. The product may be differentiated if students are allowed to present their work in multiple ways and to receive credit for diverse skills that are evident in the completed product. Finally, the learning environment may be differentiated by providing quiet space in the classroom for some students, being explicit about how help may be accessed if a student is stuck, and making available culturally familiar support materials to students as necessary.

The twin lenses provided by combining insights from Stein and Smith (1998) and Tomlinson (2000) will help to identify steps taken by the teacher to address both goals of instruction: productive struggle and differentiation.

Method.

Context.

The study is based on a mathematics laboratory that took place over four days (approximately eight hours) in July 2018. The classroom was convened specially to provide mathematics teaching for students, with a focus on differentiated instruction and challenging all students. Furthermore, the laboratory was a site for professional development with over 20 researchers, teachers and teacher educators observing the teaching and subsequently discussing it. Finally, the laboratory was a site for research with extensive records of practice being collected, including video recordings, student work, other classroom artifacts, and teacher planning notes.

Participants.

Twenty-four children – 10 girls and 14 boys – were in the class which lasted for two hours per day over four days. The children had completed fifth class in 11 different schools. Parents and guardians applied for places in the school for their children for multiple reasons, such as their child finding mathematics difficult, their child liking mathematics, or because a friend was attending. In other words, although parents opted to send their child to this class, a wide range of levels of mathematics and motivation could reasonably be expected among those participating.

The teacher has substantial experience as a primary teacher (11 years) and a primary mathematics teacher educator (19 years). He works as a researcher on a project that is focused on mathematics instruction that is differentiated and challenging for students. For several years he has taught children mathematics in a laboratory format as a form of professional development for teachers but this was the first time that the focus was explicitly on differentiation and challenge.

Over the four days of the laboratory, the students worked on four mathematical tasks, each for two hours. One was based on volume, two on fractions, and one, the subject of this paper, on algebra (see Figure 1).

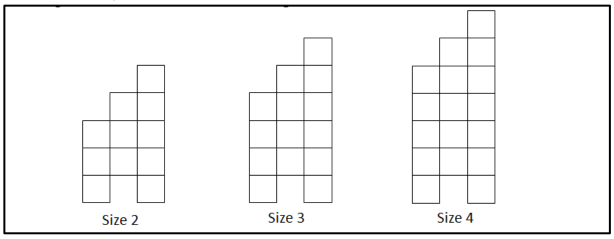
Data.

Four sources of data were used in this study. First, the task itself, which went through several iterations before being used. Second, the lesson plan that was used as the basis of the lesson. Third, a video recording of the lesson as it was enacted will be analysed. Finally, students' work on the task will be presented.

Data analysis.

Open coding in which data are scrutinised and concepts identified (Corbin & Strauss, 2008) was used for analysing the four data sources. This involved closely and repeatedly reading the written materials and closely watching and re-watching the video data. The second author identified initial categories and themes, using methods outlined by Corbin and Strauss (2008). Although the coding was open, in that the data were decomposed and concepts were sought and named, the coding complemented a deductive approach to data analysis (Gale et al, 2013) which was informed by the frameworks of Stein & Smith (1998) and Tomlinson (2000).

Alex uses identical tiles to make different sized chair designs for a school art project. The pictures on the sheet show the first three designs created, size 2, size 3 and size 4. Alex wanted a *rule* that would help work out the number of tiles needed for a chair of any size.



Q 1

(a) If Alex wanted to create a size 5 chair, what would it look like? Can you draw it or use other materials to represent it? How many tiles would be used?

(b) Work out the number of tiles needed for the size 6 and size 7 chairs. Explain how you did this.

(c) Draw or make the size 1 chair. How many tiles did you need?

Q 2

(d) Do you notice any pattern between the chair size and the number of tiles needed each time? Discuss this pattern with your partner(s).

Q 3

(e) Alex wanted to create a size 20 chair. Talk with your partner(s) about a *rule* that would help Alex work out the number of tiles needed for this chair.

(f) Would this rule work for the previous chair sizes?

(g) If yes, write out this *rule* in words.

(h) Discuss if it would work for a chair of any size.

Q 4

(i) Could you re-write this rule using symbols/letters?

Q 5

(j) Use the *rule* to calculate the number of tiles needed for a "size 50" chair?

Figure 1: Algebra task (without enablers or extensions)

Findings.

In terms of the framework/task analysis guide by Stein & Smith (1998), the chosen task is deemed to make "higher level demands" on learners and it has characteristics of both procedures with connections (e.g. use of a table for the first bullet point; writing a rule for the second; getting students to compare solutions for the third) and doing mathematics.

In analysing video recordings of the two hours of teaching, three themes were identified: Teacher actively guiding the students, teacher establishing classroom norms that facilitated differentiation and work on challenging tasks, teacher facilitating students to be resources for each other's learning. Each of these themes will now be elaborated on in more detail.

Teacher actively guiding the students.

Six ways in which the teacher actively guided the students were identified by analysing the tasks used, the lesson plans and the video. The first related to the choice of task and planning. Each task was chosen carefully so that all students had the opportunity to make some progress on it and some students could complete it all. For example, the basic algebra task (See Figure 1) was complemented by preliminary "enablers", for students who struggled to start or make progress on the task and more challenging "extensions", for students who successfully completed the basic task. One enabler suggested organizing data in a table and another commenced a rule pattern which could be continued. An extender presented for comparison three different methods used by other "students" to find the rule to calculate the number of squares needed for any size chair.

During autonomous work, when two students were working on tasks, clear instructions were issued to students. For example, the teacher suggested that "to make things easier for yourselves, make sure you label the different chairs."

The teacher deliberately directed students' attention to particular ideas or concepts. For example, at different times he found ways to introduce the idea of a row, a constant, and a variable. These concepts were introduced in response to comments or confusions from the students and would be help them in working towards a general rule.

The teacher sought opportunities to highlight ideas he believed would support students' work. As students worked on the task, the teacher asked them to pause their work and he introduced the following idea to support their work:

> Just one thing I want to show you. This is just a convention and it might be helpful for some of you. Some of you have already moved on to this step. In algebra, which is what you're doing now, this is called algebra, this kind of maths. One of the things you do in algebra sometimes is that you use letters as well as numbers so sometimes you might use x and y. Sometimes you might use a and b or sometimes you might use n." (5533 @23:32)

In Lilly's work (Figure 2), it appears from what she has written that she sees the pattern. However, although she is aware that the letters may be used, she is unable to integrate them into the general solution that she has identified. In contrast, Abby was able to express one general formula that could be used to calculate the number of tiles in a chair, where y is the size of the chair and y x 3 +5 gives the number of tiles (Figure 3). Note that Abby also uses the term “constant” in her written answer.

The teacher questioned students for clarification. For example, using Lilly’s idea of a chair of size 0, he asked “Can anyone explain what Lily has just said or put it into your own words?” (5527@8:40). On another occasion he used revoicing (O’Connor & Michaels, 2007), when he attempted to clarify what a student was saying, “so you’re saying the shape is always the same except for….” (5534@19:12).

The teacher attempted to increase the challenge for students. For example, he asked if anyone could come up with a rule that works for any size chair (video clip 5527, 5:03) and on another occasion he set the following challenge for students: “Now I want everyone to think carefully. How does David’s rule compare with Tim’s rule? (Clip 3527 @ 15:55).

Finally, the teacher’s active guiding of students involved giving them time to think. After posing a question, he allowed students sufficient time to think of an answer before calling on someone to answer.

Teacher establishing classroom norms that facilitated differentiation and work on challenging tasks.

A second theme identified relates to how the teacher established and promoted classroom norms that facilitated differentiation and work on challenging tasks. One such norm that was encouraged was that confusion is okay on the path to learning/understanding. At one point the teacher said “Remember it’s really important to say that you’re confused. Have you followed anything at all of what Tim has said?” (5527@11:05). This followed a contribution from Tim that would lead towards a solution to the task.

Other norms evident in the video include the use of talking about mathematics. Students do not just write their solutions but they are encouraged to share their ideas with their classmates and to have them questioned and critiqued by their peers.

Students are encouraged to take responsibility for their learning. The nature of learning is discussed. For example, students are told that like working out in the gym, learning mathematics requires effort. That doesn’t mean that “maths is difficult” or you are “no good at maths” but that over time, the effort will lead to learning.

Manipulative materials are made available to students (e.g. square tiles) and students are encouraged to use what they need if using the materials will help them grasp the concept. Students are given autonomy in deciding to use or not use the materials.

Finally, students are encouraged to represent their solutions in visual form. In particular, they are encouraged to explain ideas for their parents using representations. This differs from a more common approach in schools where students simply present their solutions in their own notebooks in numerical form.

Teacher facilitating students to be resources for each other's learning.

The third theme identified from the video data was the teacher's encouragement of students to be resources for one another's learning. At the outset of the summer school the teacher explicitly stated to the students that "you can learn a lot from your classmates." The idea of students being resources for each other's mathematics learning was encouraged throughout the summer school.

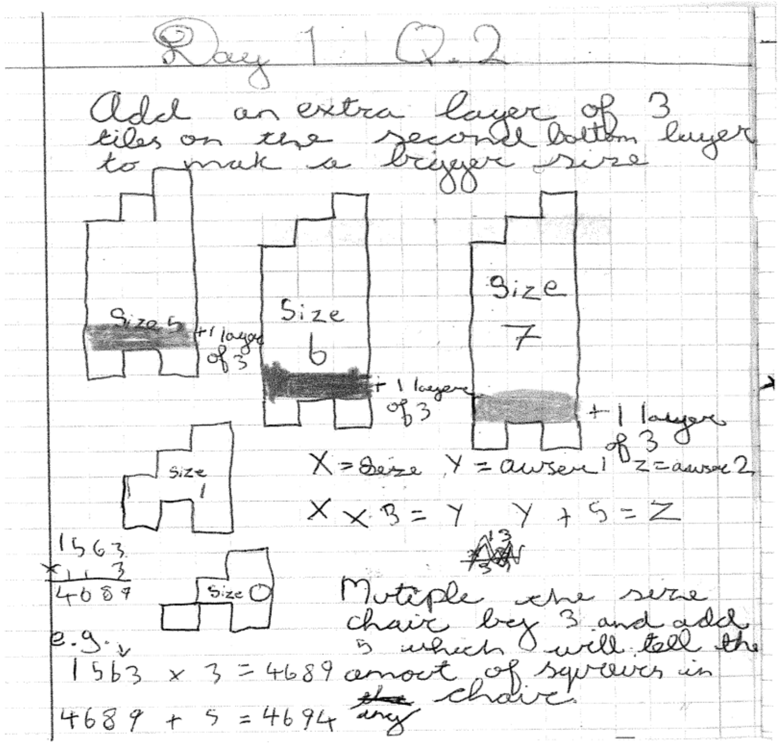


Figure 2: Lilly's work on the algebra problem

One way the teacher did this was to ensure that students or groups of students were given opportunities to share their solutions. Attempts were made to sequence the sharing in a way that progressed from more naïve to more sophisticated solution strategies.

Students' sharing of solutions was complemented by several examples of the teacher asking students to repeat, revoice, or explain what was said. This had the potential to amplify what students said (by hearing them a second time), to clarify what was said (through explanations) and it encouraged students to listen to one another because they realise that they may be asked to respond to another student's input.

Students were required not only to listen to contributions from their classmates; the teacher frequently asked students to analyse and compare different solutions and ideas. This affirms students' solutions but it further has the potential to help them see where some solutions might be more efficient than others.

Finally, students are frequently asked to clarify their ideas or those of their peers by using words and representations. For example, on one occasion when students were working on the task, the teacher asked a student to “Come up and show us how you laid it out in your notebook” (5527 @9:20).

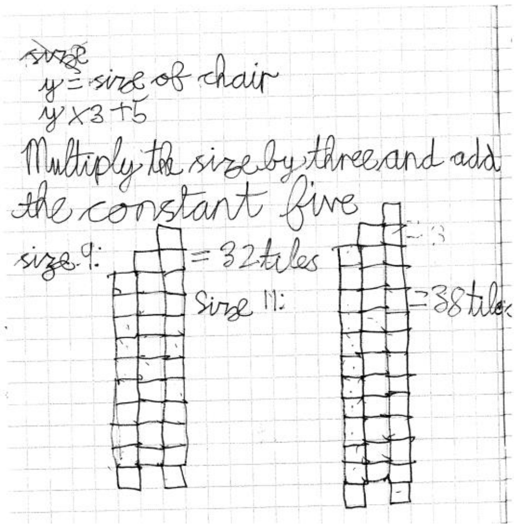


Figure 3: Abby’s work on the algebra problem

Discussion/Conclusion.

We return now to the research question that we posed at the start of this paper about how the teacher supported differentiation and maintained or modified the challenge of the task. Modification of the task was planned in advance through the use of enablers and extenders. In the course of the lesson three themes were identified to capture ways in which the teacher implemented differentiation and cognitive demand. He actively guided the students to take responsibility for working on the task. He established and maintained classroom norms that supported these aims and he encouraged students to act as resources for each other’s learning.

What these findings suggest is that through intentional and deliberate actions, a teacher can combine both differentiation and challenge in mathematics instruction. The need for differentiated instruction may be imposed on a teacher by the different rates at which students complete tasks. However, in order to engage students in challenging tasks, teachers must set that as a specific instructional goal and must have the required knowledge to implement it (Henningsen & Stein, 1997). Although the importance of teacher guidance and establishing appropriate norms have been identified as factors in supporting the use of challenging tasks independently, the additional factor of students acting as resources for one another was found to apply when challenge was combined with differentiated instruction.

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