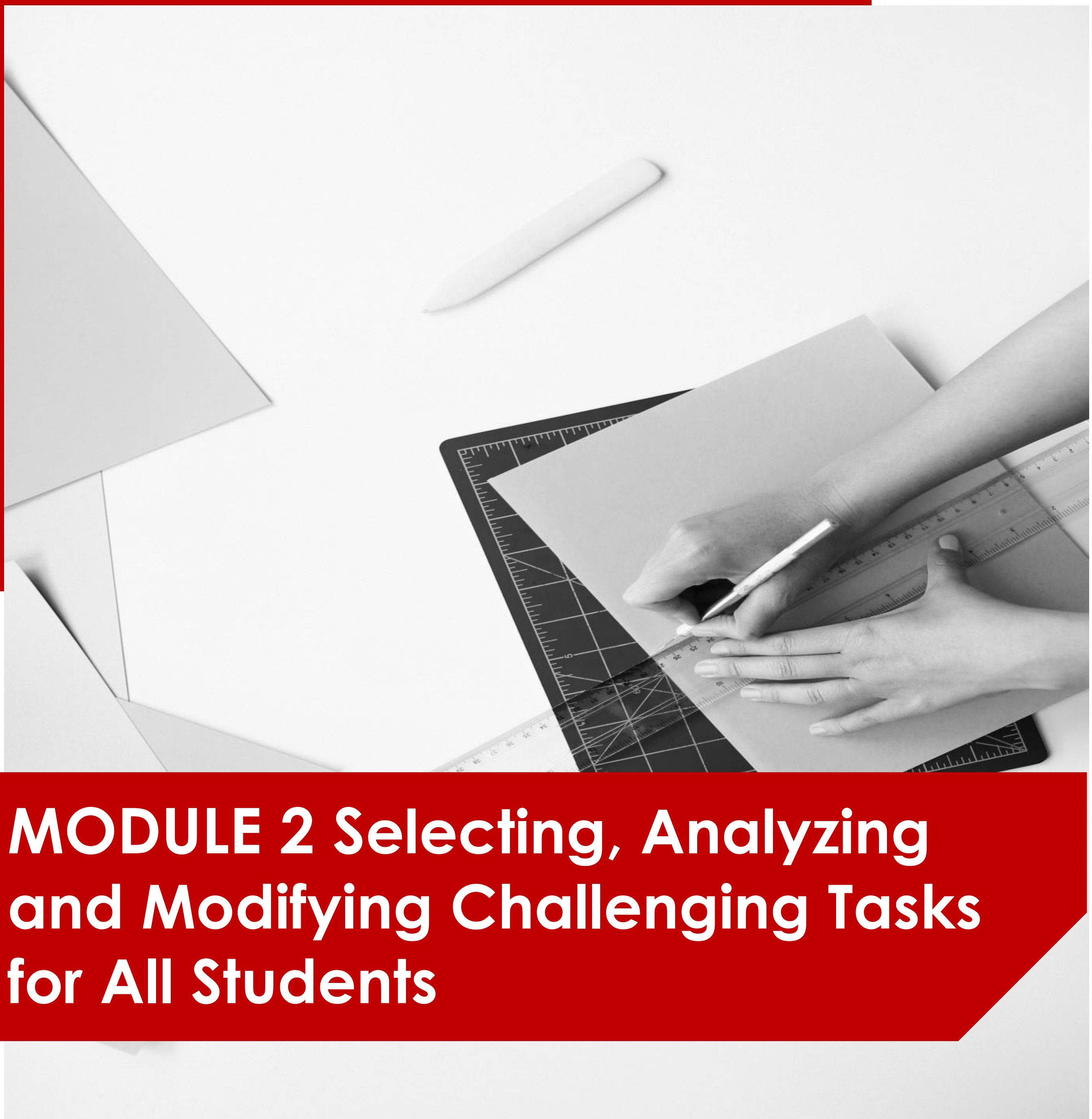

MODULE 2 Selecting, Analyzing and Modifying Challenging Tasks for All Students

EDUCATE Project

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SYMBOLS

Next to each activity there is one or more of the following symbols:

Individual work

Video-club setting

Read

Write or complete

Link-to-File

Watch

Reflect

Discuss

Learning Objectives

Plan

Assess

CASE OF PRACTICE 2

Planning for Differentiation: Considering the Task for Different (Groups of) Students

Overview

CONTACT HOURS	2 hours and 30 minutes
TYPE OF RESOURCES	Post-it notes; Videoclips; Interview segments; Tasks; Lesson plan extract
EMPHASES	What makes a task challenging for different groups of students Adjusting/Modifying a task to make it more or less challenging for different groups of students

Activities

Opening Activity

Post-it® Parade

You are provided with a few post-it notes. Write one idea/ issue/ concern/ question per post-it note answering one of the questions below, post the post-it notes on the wall and discuss them with your colleagues.

- What have you learned from the previous meeting?
- Are there any questions to clarify, issues you think were left unresolved, or ideas, concerns or positions not yet considered?

Activity 1 - Analysing Practice

Video club Component

In the previous case of practice, you were asked to (a) **select** two tasks from your own textbook, one mathematically challenging and one less mathematically challenging; (b) **plan** and **videotape** a lesson during which you would implement the mathematically challenging task; and (c) **watch** and **determine** at which level the task was implemented.

Share with your colleagues the episode you selected from your videotaped lesson in which the level of mathematical challenge was either maintained or adjusted. Explain what this episode is about and your rationale for selecting it.

Discuss the shared episodes with your colleagues:

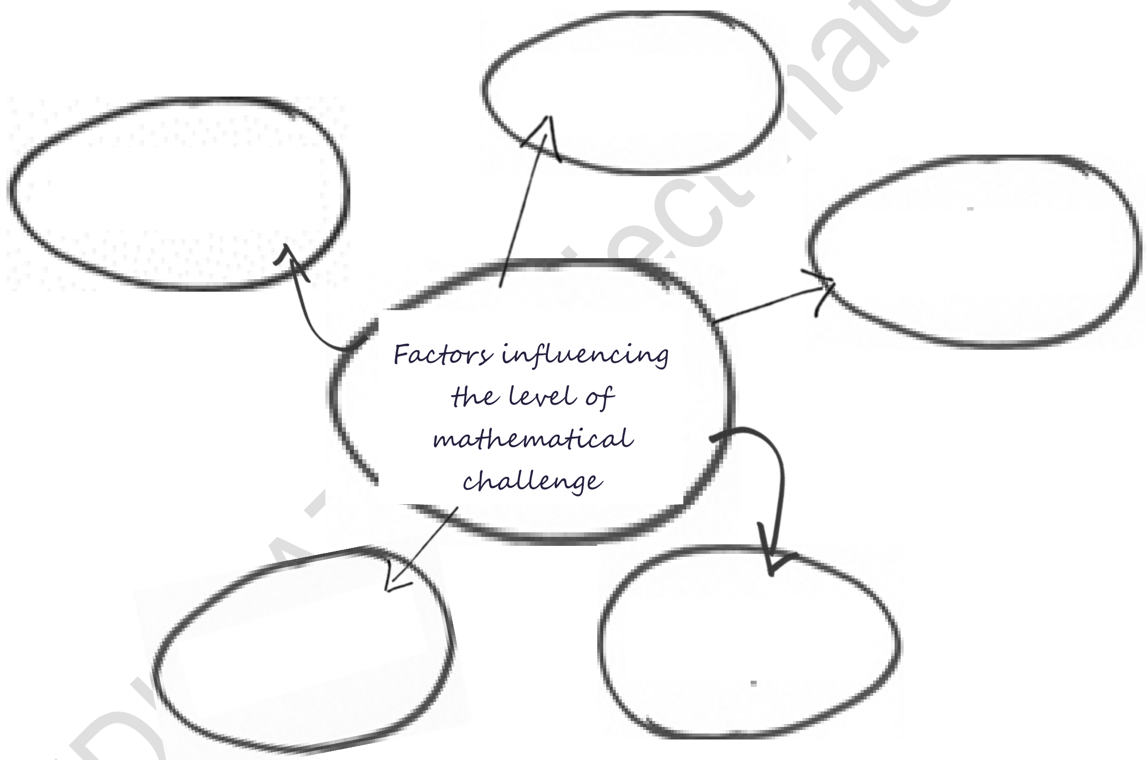
- Describe what the teacher and the students were doing during this episode.
- Were there cases in which you adapted the level of mathematical challenge? What informed these decisions?
- Do the mathematically challenging tasks work in the same way for all students?
- Did the implementation of the tasks unfold exactly as you planned it?
    - If not, what changed during the implementation of the two tasks compared to how they were planned?

The level of demand of mathematical tasks could change during their implementation in mathematics lessons. Hence, the tasks as presented by the teacher or as worked on by the students might be different in terms of their mathematical challenge compared to how mathematically challenging the task you selected for your teaching was. As discussed in Module 1, this could greatly impact students' learning opportunities. So, it is important to focus on different reasons/factors that might inform these decisions.

Activity 2 – Considering Factors Influencing Task Implementation

Brainstorming Activity

Based on previous lessons you have taught and also considering the following excerpts from interviews we conducted with secondary classroom teachers when discussing the challenges they faced in implementing mathematically challenging tasks with all their students, list a number of factors that may affect the level of intended mathematical challenge of the task(s) as planned during the task presentation and implementation.



You know what? I sometimes feel that we are rushing children through lessons rather than focusing on engaging them in procedures of reasoning and understanding. Having to teach an overcrowded curriculum, especially in Grades 11 and 12, which I am teaching these last five years, and trying to follow the guidelines of the Department makes me feel under constant pressure to get students to reach certain benchmarks that don't necessarily match the way they learn.

Margaret

Carlos

What can I do when half of the students have finished the task, others try it for a couple of minutes but then they give up, two or three students try really hard but they still don't reach a solution, and the rest of the students have only just begun? You can tell that the more capable students are getting bored if I spend time explaining more to the less capable ones. What's the point of instruction for those students if they already know the content or if they have already got there and they want to move forward?

Although we recognise all these student differences, to simplify matters we will suspend issues that relate to students for the time being. We will assume that when solving a task, at least three different groups of students can be identified: high-achievers, mid-achievers, and low-achievers. In the remaining activities of Case of Practice 2, we will consider how, as teachers, we can plan in ways that can help us scaffold these different student groups without diminishing the mathematical challenge of the tasks.

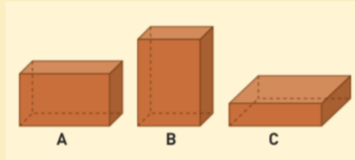
Activity 3 – Planning for the Use of Enablers and Extenders

Below, you can find the task "The Boxes" which comes from a Portuguese tenth-grade classroom. The narrative that follows gives an overview of students' autonomous work while working on investigating the maximum number of boxes that can be put into the container. Read the task and the narrative and then consider the questions that follow.

<u>Task 1</u> ('The Boxes', Geometry and Measurement, Grade 10)

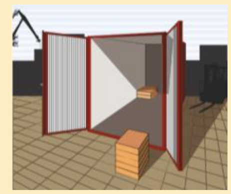
1. Consider one container with a width of 2m, a length of 4m and a height of 2.5m to transport boxes with the shape of rectangular prisms and with the following dimensions: length 70cm; width 50cm and height 30cm.

   a) Suppose that the boxes can be introduced in the container in any position, as the figure shows:



If all boxes are packed as in position C, investigate the maximum number of boxes that can be put in the container. Show how you got your answer.

b) If all boxes are packed in the same position inside the container, investigate which of the given positions A, B, & C should we choose in order to transport the maximum number of boxes. Show how you got your answer.



Narrative (The Boxes Episode)

The teacher's goal for this lesson was to engage students in a problem about different ways of packing objects, involving three-dimensional geometry with a high emphasis on visualization and measurement. The task required students to mobilize their knowledge about the volume of parallelepipeds, which was covered in last year's curriculum. The episode recounted below occurs as the teacher approaches some groups of students while they are working on the task in small groups.

Mr. Manuel: How are you doing here?

Fernanda: Sir, this is for primary school students! [laughing] We know this from a previous year. First, we calculated the volume of the container which is 2x4x2.5=20 and then we calculated the volume of a box, which is 70x50x30=105000. Then we… divided 20/105000 which is [she types it on her calculator] 0.00019047619 to find the total number of boxes.

Mr. Manuel: Hm… Nice way of thinking. But is your answer reasonable?

António: I told you, Fernanda!

Mr. Manuel: What is wrong, António?

António: We didn't consider that the dimensions of the boxes are in centimetres!

Mr. Manuel: Hm... Is that the only problem?

Fernanda [seeming somehow disappointed]: Well, I am tired. We had gymnastics during the previous lesson hour [implying that this is why she is tired now].

Mr. Manuel [trying to encourage Fernanda]: Do not give up, you did a really nice job! Reread the instructions carefully and I will be with you in a couple of minutes.

The teacher decides to go to another group of students who seemed to be discussing something irrelevant to mathematics.

Filipe: Sir, we're done!

Mr. Manuel [quite surprised]: Have you already finished both parts? Talk to me about your work.

Filipe: We began by finding… computing the volume of the container and the volume of the box. We did this to figure out how many times the volume of the box fits in the container.

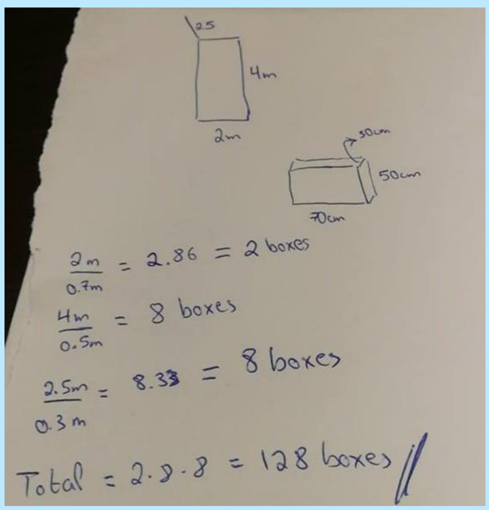
João: At first, we thought that we were done and we proceeded to question B. But, fortunately, Teresa noticed that there was a trick!

Mr. Manuel: Really? And what was the 'trick', Teresa?

Teresa: The position matters.

Mr. Manuel: The position of what?

Teresa: In which we put a box inside the other [meaning inside the container]. For example, position C is different than position B, can you see? [points to the picture of question A and the drawing she made on her handout].



Teresa's handout

Mr. Manuel [quite pleased]: Interesting! Let's see what your classmates did; we will shortly share our work in the group—so, Teresa, and the rest of the group, get prepared to explain your thinking.

The teacher moves quickly to yet another group of students who seem frustrated.

Miguel: We don't know how to start!

Mr. Manuel: Why? Do you have an idea? I would love to hear it! It doesn't bother me if it is wrong.

Catarina: Not really…

Mr. Manuel: OK, why don't you go to Filipe's table to see what they did? We will discuss the problem together in plenary in a couple of minutes.

Guiding Questions

- What is the level of mathematical challenge of the 'The Boxes' task?
- How did the teacher manage task complexity?
  - Was his management of task complexity effective? Why? Why not?

In another classroom, the same task was used again by a tenth-grade teacher. In this case the teacher planned to use *'Enablers'* and '*Extenders'* as a way to manage task complexity. Below you can find the task and an excerpt from his lesson plan, which was edited to focus on the differentiation strategies this teacher used. Read the task and the lesson plan excerpt and then consider the questions that follow.

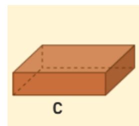
Lesson plan Extract for 'The Boxes' task

Tasks & Learning Activities	Expected Duration	Differentiation
Solving question a (Autonomous work - Mixed ability grouping will be the default arrangement, but teacher discretion also applies)	10 mins	• Introduce **Enabler 1** as required to students who do not know how to start to represent the situation with a model and understand that they have to take into account the position of the boxes in the container, and finally, realize that they can follow a strategy. **Task Enabler 1** Use the materials which can be found in the boxes under your desk. Create a physical model to represent how the boxes will be put into the container at the three different positions: • Position A • Position B • Position C How does the position of the boxes influence the number of boxes that can be used to fill the container? • Monitor use of **Enabler 1** and introduce **Enabler 2** to students who face a difficulty taking into account that the boxes will be put one next to the other and also one over the other. **Task Enabler 2** Find how many boxes can be put at the base of the container. • Monitor use of **Enabler 2** and introduce **Enabler 3** to students who consider using the volume of the rectangular prism and

do not pay attention to the restriction put by the linear dimensions of the boxes (do not consider the need to take into account that they should consider only the integer part of the linear measurements for width, length and height).

Task Enabler 3

Write down the dimensions of the box if we place it at position C and the dimensions of the container.

Position	Height	Length	Width
			
			

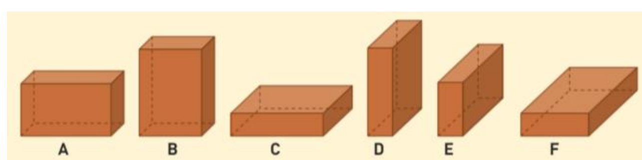
1. What do you notice? Explain your thinking.
2. Do you figure out any relationships among the dimensions of the boxes and the container?

Extenders 1 & 2 (for those who complete questions a & b)

N/A

1. During the autonomous work for questions a and b, students who quickly and accurately complete questions 1 - 5 (likely without need for the **Enablers 1 & 2**) and prove the authenticity of their solution path, can immediately be given **Extenders 1 & 2.**

Task Extender 1



Apart from the given positions A, B & C, investigate if the given positions D, E, & F should be chosen to transport the maximum number of boxes. Consider that all boxes are packed in the same position inside the container. Show how you got your answer.

Task Extender 2:

What should be the dimensions of the box so that it fits exactly into the container not leaving any gaps when placed in:

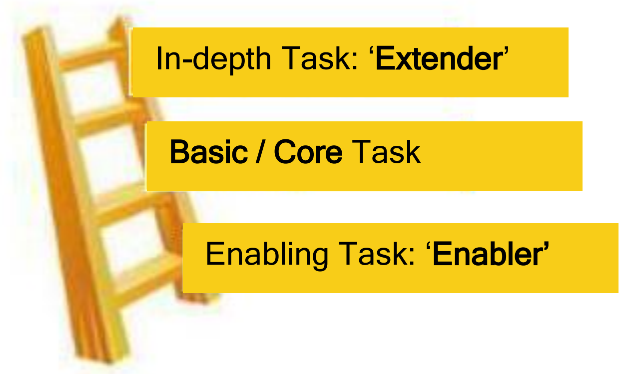
i. Position A?
ii. Position C?
iii. Position D?
iv. Any position?

Guiding Questions

- Focus on the ‘*Enablers*’ and the ‘*Extenders*’ used by the teacher.
  - How did the teacher plan on using them?
  - Consider their contribution to handling task complexity. How can the second teacher’s lesson design help better handle task complexity during task presentation and implementation?

The Ladder of Task Differentiation

Students do not necessarily experience the same level of mathematical challenge when they are engaged with the same mathematically challenging tasks. Some tasks can be too difficult to solve or too easy for different students. At some point, students may need differentiated instruction based on their particular learning needs. One way to deal with this complexity is to develop and use ***tiered tasks***. Tiered tasks are a series of related tasks of varying complexity which *focus on the same content or curriculum objective*. Often times, as teachers, we prepare one task to engage all our students in mathematically challenging work—this task represents what our curriculum is asking us to do, often for the average students—we will call this “***core task***”. However, some students (not necessarily the same each time), might need to work on a task that provides more support or scaffolding, we call these tasks ‘***Enablers’***. Other students (again, could differ from task to task), might finish the core task early and seek additional mathematical challenge; this challenge can be offered by tasks called ***Extenders***. *Enablers* can ‘enable’ students thinking when they need extra support or guidance with the *Core* task to proceed; *Extenders* can ‘extend’ early finishers’ thinking. “A good way to visualize a tiered task is the image of a **ladder”**, where a Basic/Core task “appears on the middle rung, the advanced version of the core task”, the extender, on the top rung and another modified version of the basic/core task, the enabler, “on the bottom rung” (Primary Professional Development Service, n.d., p.13).



Source: adapted from Primary Professional Development Service (n.d.), Differentiation in Action! http://www.pdst.ie/sites/default/files/Session%202%20-%20Differentiation%20Resource%20_0_0.pdf

You are planning to teach the following mathematical tasks next week and you want to think of ways to differentiate the tasks so that all your students will participate and learn. First identify what makes each task mathematically challenging (or less challenging) and then consider different ways of differentiating the tasks at least one level up (extenders) and one level down (enablers) without necessarily presenting your work in written form.

Task 1 ('Charlie's number trick', Grade 7) Watch the video to see Charlie's number trick (https://nrich.maths.org/7208). **Video Transcription:** Charlie said: "Alison, think of a two-digit number. Reverse the digits and add your answer to your original number. I bet your answer is a multiple of 11." Alison chose 42, added 24 and got the answer 66: "It is! How on earth did you know that?" Charlie said: "I'm not sure. Let's try to work it out." a) Can you explain how Charlie's number trick works? b) Charlie came up with his own explanations:	Level of Mathematical Challenge: Possible Enablers: Possible Extenders:

Charlie imagined a two-digit number *ab*, where *a* represents the number in the tens column, and *b* represents the number in the units column. This can be written as $10a+b$. Similarly, *ba* can be written as $10b+a$. Charlie added these together to get $11a+11b$, which he wrote as $11(a+b)$.

Here are some similar number tricks. Can you use Charlie's representation to explain how they work?

- Take any two-digit number. Add its digits and subtract your answer from your original number. What do you notice?

- Take any three-digit number. Reverse the digits and subtract your answer from your original number. What do you notice?

Source: https://nrich.maths.org/7208 (adapted)

Task 2 ('Quadrilaterals from Midpoints' Grade 10)

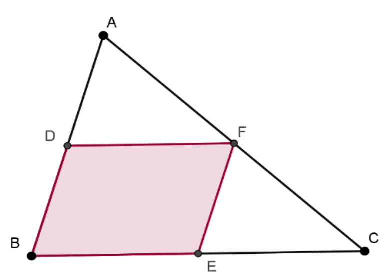


Figure 1

1. In Figure 1, the points D, F and E are midpoints of the sides of the triangle ABC. Investigate what type of quadrilateral is BDFE. Study how the quadrilateral BDFE changes when the triangle ABC changes. That is how the type of quadrilateral is connected to the type of the triangle.

Level of Mathematical Challenge:

Possible Enablers:

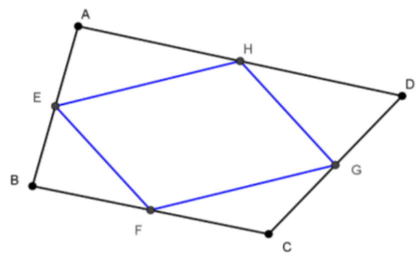


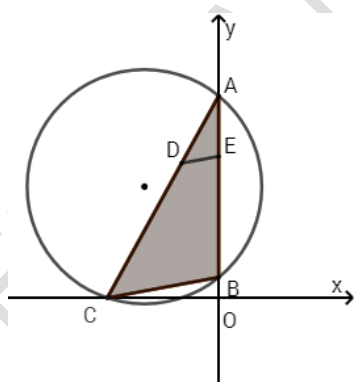
Figure 2

2. In Figure 2, the points E, F, G, H are midpoints of the sides of ABCD. Investigate what type of quadrilateral is EFGH? Study how the quadrilateral EFGH changes when the quadrilateral ABCD changes. That is how the type of EFGH is connected to the quadrilateral ABCD).

Possible Extenders:

Task 3 ('Triangle Inscribed in Circle', Grade 10)

In the plane with a Cartesian coordinate system, consider the circle with equation $(x+2)^2+(y-3)^2=10$ and the triangles [ABC] and [AED], as depicted in the figure below.



We know that:

- Points A and B belong both to axis *Oy* and to the circle;
- Point C belongs both to axis *Ox* and to the circle;
- $[DE]$ and $[CB]$ are parallel line segments and $\overline{DE}=\frac{1}{3}\overline{BC}$.

Level of Mathematical Challenge:

Possible Enablers:

Possible Extenders:

1.1. Determine the area of the triangle [ADE]. Show how you have found the answer: you should explain your reasoning and present all computations you have made. 1.2. Define analytically the triangle [ADE]. Show how you have found the answer: you should explain your reasoning and present all computations you have made.	

Differentiated Instruction: Some Principles When Modifying Tasks

When designing tiered tasks, it is important to consider at least three groups of students: those who are at the introductory level; those at the standard level; and those who are capable of more in-depth, higher-order tasks. Bear in mind, though, that these groups are not constant. A student might be clustered in the first group for one task and in the second, for another.

Using the ladder of task differentiation can be effective if the following principles are taken into account:

- Tasks should focus on learning objectives and essential concepts.
- Tasks ought to respond to the specific learning needs of different groups according to ability, readiness, degree of support required and learning preferences.
- All tasks should be engaging, active and interesting
- Extender tasks should not be just "more work" and enablers should not represent "dumbed down" versions of the core task.

Source: adapted from Primary Professional Development Service (n.d.), Differentiation in Action! http://www.pdst.ie/sites/default/files/Session%202%20-%20Differentiation%20Resource%20_0_0.pdf

Connections to (my) Practice

Design a lesson plan that includes one mathematically challenging task and 2-3 differentiation strategies from those discussed at today's meeting. Then videotape the enactment of these tasks.

Watch the videotaped lesson and determine how effective your differentiation approach was.

- In what ways did planning the lesson as discussed today help your teaching?
- Did your differentiation strategies help all students work productively on the task?
- What problems did you encounter during the task enactment? In hindsight, in what ways could you deal with these problems?

Select episodes illustrating effective differentiation up and down to share with your colleagues during the next meeting.

Closing activity

Working in pairs name some differentiation strategies you have considered for adjusting the level of mathematical challenge of a given task.