TEACHER EDUCATOR MODULE 2
Selecting, Analyzing and Modifying Challenging Mathematics Tasks

EDUCATE Project

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Organization	People[1]
University of Cyprus	Dr. Charalambous Y. Charalambos Professor Constantinou Constantinos Georgiou Chloe Georgiou Kassandra Kasapi Evridiki Professor Koutselini Mary Dr. Olympiou George Dr. Philippou Stavroula Professor Pitta-Pantazi Demetra
Institiúid Oideachais Marino, Institute of Education. An Associated College of Trinity College Dublin, the University of Dublin	Burke Damien Concarr Ann Dr. Delaney Seán Dr. Gurhy Ann Marie Dr. Prendergast Mark Purtill Trevor Timmins Paul
Hellenic Republic, National and Kapodistrian University of Athens	Professor Potari Despina Dr. Psycharis Giorgos Dr. Triantafillou Chryssavgi Professor Zachariades Theodossios
U Lisboa, Universidade de Lisboa; ie Instituto de Educação	Professor da Ponte João Pedro Dr. Guimarães Henrique Dr. Henriques Ana Dr. Santos Leonor Dr. Oliveira Hélia
Cyprus Pedagogical Institute	Dr. Agathangelou Sofia Dr. Christofidou Elena Dr. Papadouris Nicos
Ministry of Education, Culture Sport and Youth	Demosthenous Christos Ioannides Stelios Dr. Kythreotis Andreas Dr. Savvides Yiannis Dr. Stylianides Marios Dr. Theodorides Andreas Theodorou Rodoula Dr. Yiallourides George
	Dr. Michaeloudes George Nicolaou Savvas

[1] All names are listed in alphabetical order.

CASE OF PRACTICE 2

Planning for Differentiation: Considering the Task for Different (Groups of) Students

Expected outcomes

By the end of this Case of Practice, teachers will be able to:

- Identify and explain what makes a task challenging for different groups of students (LO2)
- Adjust or modify a mathematics task to make it more or less challenging for different groups of students (LO3)

Short Description of the Activities and How They Can Be Enacted

Opening Activity

Indicative Duration: 15 minutes

Post-it Parade

This activity is a good way for you to get a general sense of what sort of questions, concerns, ideas, or issues the participants may have from the previous Case of Practice. Students are provided with two questions/prompts for which they need to generate ideas. To enact this activity, give each participant a few post-its (the number of post-its depends on the available time), and have them write out one idea/issue/concern/question per post-it. Participants then post the post-its on the flipchart or wall. Depending on the question or prompt, it may be useful to have the participants place the post-its in areas grouped by topic/concern. The goal behind this activity is to help teachers consider issues, other than task selection, which are important for a teacher to take into account during lesson planning.

Activity 1 - Analyzing Practice

Indicative Duration: 60 minutes

Videoclub Component

In the previous session, teachers were asked to select two tasks from their own curriculum materials, one mathematically challenging and one less mathematically challenging, plan and videotape a lesson during which they implemented the mathematically challenging task; and watch and determine at which level the task was implemented. The teachers were also asked to select short clips (typically 3-5 minutes each) in which the level of challenge was either maintained or adjusted. At least a week before today's session, the teachers should have sent via email their videotaped lesson via 'We Transfer', as well as, the selected task, the timestamps of the videoclips they believe capture the focus of the activity, and a short note explaining their rationale for selecting these episodes. Because this is the first time in Module 2 teachers share clips from their teaching, the following detailed instructions for teacher educators (also discussed in Module 1) are helpful in order to make the first video-club session a positive experience for teachers:

- Activity 1 of Case of Practice 2 is a good opportunity for you to check teachers understanding about the key ideas discussed in the previous session (i.e., select mathematically challenging tasks; analyzing the tasks and identifying what makes them mathematically challenging using the TAG; and comparing the level of mathematical challenge of a task as it is presented in the curriculum materials to the level of mathematical challenge as it is enacted during teaching) and for teachers to develop a 'professional vision' by noticing and interpreting classroom interactions. You can use the guiding questions of the opening activity to help teachers initiate a discussion around the shared episodes.
- **Before today's session**, watch carefully the videotaped lessons of all participants, read the materials sent to you by the teachers and decide which videoclips can initiate rich discussions around the issues discussed in the previous session. While watching the clips you could keep notes on interesting teacher actions/practices and classroom interactions that occured during teaching. The selection of the clips is really important and should be done deliberately based on the goal of the activity. You need to be careful with the selection of the clips so that the selected clips help you raise key issues and practices that emerge from the videotaped lessons. For instance, a teacher might have suggested clips that will help you surface important ideas and key practices for maintaining the level of challenge; another teacher might have suggested clips that do not serve the purpose of the activity. Value teachers' choices or concerns by including some clips they suggested but remember that the goal is to focus on specific teaching practices that were employed during that episode, consider their influence on students' opportunities to learn, and make suggestions for improvement.
- Because this is the first time in Module 2 that the teachers share videoclips of themselves teaching, **your role as a teacher educator is extremely crucial** for how the next video-club sessions will unfold (See General Guidelines for more information). Teachers must feel comfortable to share their clips with the whole group and in no case feel disapproved by their colleagues or you if something in their lesson did not work as expected. For instance, teachers might select a clip in which the level of challenge was decreased or a clip in which they believe that they maintained the level of challenge at a high level but this was not actually the case. In such cases, you need to be careful not to offend teachers, but to discuss the clip with **a focus on teaching, rather than the teacher**. Hence, it is really important to begin this activity by

reminding teachers what was also discussed in Module 1: that your goal is to **use the videotaped clips as records of practice**, which will help you better understand and learn from real classroom interactions and improve your work. Ask them to **refer to teachers in general** without naming the particular teacher and recognize that there is no such thing as perfect teaching. Remind teachers that your intention while watching the clips is to start noticing and interpreting different significant decisions and actions that worked well or could lead to improved interactions among the teacher and the students, rather than evaluating the lesson and the teacher. A good way of making teachers feel comfortable is to begin with sharing a clip of a teacher who feels more confident with sharing his/her teaching (especially if the group of teachers taking Module 2 is different from the group of teachers who took Module 1). Moreover, teachers should begin by referring to the positive features of teaching and then explaining what could be improved acting as critical friends. You may ask teachers to say something they found positive or helpful in the clip and why they feel that way and/or mention a question occured to them while watching it. Do not forget to allow teachers time for several responses to each of your inquiries, including a chance for them to propose answers to their colleagues' questions.

- During the first video-club session, it is also particularly important to **remind teachers of the commonly agreed rules regarding whose videoclips and in which order will be presented** that were discussed in Module 1. For instance, you can agree to rotate so that in every next meeting at least one clip from each teacher will be viewed.

Activity 2 – Considering Factors Influencing Task Implementation

Indicative Duration: 15 minutes

In this activity, teachers are asked to (a) consider previous lessons they have taught as well as two excerpts from interviews the EDUCATE project team conducted with primary classroom teachers during the first phase of the project and (b) list a number of factors that may affect the level of intended mathematical challenge of the task(s) as planned during task presentation and implementation. The guiding questions of the previous Activity can help you in making a smooth transition from the Videoclub Component to Activity 2. For example, in the previous activity, participants could consider whether the implementation of the tasks unfolded exactly as they planned it or not and then in this activity, suggest some factors influencing the level of mathematical challenge. Give participants 4-5 minutes to read the interview excerpts and work either individually or in pairs; then, ask them to share their responses with the whole group. During this sharing, you might consider creating clusters of factors (e.g., student related factors, teacher related factors, classroom factors, external factors, factors related to student autonomous work or whole-class discussion, etc.) and listing them on a flipchart as participants share their ideas and examples.

Teachers could possibly refer to some factors that influence the level of mathematical challenge, including but not limited to:

- Dealing with heterogeneity
- Students' different readiness and/or ability levels, interests, and learning styles
- Different learning paces within the same classroom
- Students' prior knowledge (some students do (not) have the necessary knowledge to proceed to the knew knowledge)
- Handling classroom management problems
- Organizing the classroom in a proper way and overcoming existing/well-established classroom norms
- Dealing with time constraints
- Pressure to cover an overcrowded curriculum
- Guidelines from the Ministry/Administrators on what to teach and how to teach it
- Certain benchmarks that students have to reach by the end of the school year
- Predicting students' difficulties
- Recognizing the challenging elements of the task(s)
- Teacher's limited (content and/or pedagogical) knowledge or experience
- Monitoring all students' group work / being responsive to students' ideas and offering appropriate support; striking an appropriate balance between offering guidance and offloading the responsibility for making the actual thinking to students
- Adjusting planning to students' needs
- Selecting and sequencing students' strategies / steering the discussion in a productive manner; this requires making decisions as to what to ignore, merely acknowledge or built upon

Following that, say that handling this complexity is not an easy endeavor but it is feasible when considering and trying to handle one or two factors at a time; also, not all factors apply to your classroom and teaching reality. Being cognizant of these factors that influence the level of mathematical challenge and organizing them into groups that reflect certain key components that influence student work and thinking (e.g., student factors, contextual factors, teacher factors, etc.) is a first step to start handling their influence. The important take-away from this activity is that these factors should not be underestimated because one way or another they can influence what students learn. Assure teachers that these factors, especially those related to the students will be considered in more depth in the next case of practice. Emphasize, however, that it is also important to consider the task itself and the affordances of the task as such (or modifications thereof) has/have for student learning.

Then, let the participants know that in this Case of Practice you will suspend issues related to the students, and focus more on the task and its affordances themselves. You will assume that when solving a task, at least three different (not stable) groups of students can be identified: high-achievers, mid-achievers, and low-achievers. In the remaining activities of Case of Practice 2, you will consider how teachers can plan in ways that can help them scaffold these different student groups without diminishing the mathematical challenge of the tasks.

Activity 3 – Planning for the Use of Enablers and Extenders

Indicative Duration: 60 minutes

The focus of this section is to help teachers understand that one way to deal with the complexity discussed in the previous activity is to develop and use **tiered tasks**. As explained on page 34 of the Teacher Module 2, tiered tasks are a series of related tasks of varying complexity which focus on the same content or curriculum objective. Teachers usually ask students to engage with a '**Basic/Core Task'**; but sometimes they need to develop **enablers** to 'enable' students thinking when they need extra support or guidance with the Basic/Core task to proceed and **extenders** to 'extend' the thinking of students who need greater mathematical challenge than that presented by the Basic/Core task.

Allow teachers approximately 10 minutes to read 'The V-fomation' task and the accompanying narrative from a real teacher-students interaction during the student autonomous work phase, and consider the guiding questions that follow. Next, initiate a 10-minute discussion that focuses on the effectiveness of teacher's way of managing task complexity. The first thing teachers need to understand is the level of mathematical challenge of the task according to the TAG. Ask teachers to share their responses to this question by justifying their ideas e.g., this is a "doing-mathematics" task because it asks students to observe, identify, and extend an algebraic pattern using a real-world context (Question a); the multiple steps of the task help students to gradually develop a generalization of the V-fomation pattern (Questions b-c) and then apply this 'rule' to find the number of dots of a given term/formation or the number term/formation of a given number of dots (Questions d-f). Following that, ask teachers to consider whether this complexity was maintained during the presentation and implementation of the task. Here are some points that are worth noticing and discussing while facilitating the discussion:

- Which factors made it easy/difficult for teacher to manage complexity?
- What do you think about the way the teacher tried to scaffold Marcos/Mary/Peter?
- What do you think about the sequence in which the teacher approached different students?
- Did the teacher succeed in maintaining the mathematical challenge for all three students?
- Was there anything that the teacher could have done differently to manage complexity more effectively?

Next, teachers are provided with a similar algebraic task of an Irish teacher accompanied by a lesson plan excerpt in which the teacher planned to use 'Enablers' and 'Extenders' as a way to manage task complexity. Allow teachers 2-3 minutes to read the task and then ask them to consider the level of mathematical challenge of the task (and possibly find in which ways 'The Chairs' task is similar to the 'V-formation' task, e.g., both are highly challenging, students have to observe, identify, and expand an algebraic pattern, and generalize a rule on how the pattern evolves). Following that, ask teachers to read the lesson plan excerpt and then consider the guiding questions. First, teachers can focus on understanding how the teacher planned on using 'Enablers' and 'Extenders' (guiding question 1). For answering this question, they can focus on the description that precedes each enabler and extender and consider in what ways the enablers or extenders can be helpful for students who might have difficulties with the core task and for early-finishers, respectively. In particular you can encourage teachers to pay attention to the following points:

- Which students can benefit from the use of these enablers/extenders?
    - "Introduce Enabler 1 as required to students who do not consider ... of the data."
    - "Monitor use of Enabler 1 and administer Enabler 2 to students who ... specified chair size)."
    - "During the autonomous work for question 1, students who quickly and accurately complete questions 1 - 5 ... be given Extenders 1 & 2."
- What is the key idea with which each group of students will work?
- What should be the entry ability/readiness/knowledge level of a student who uses e.g., Enabler 1?

The second guiding question should help teachers gradually start considering the contribution of enablers and extenders to handling task complexity during task presentation and implementation. Teachers can possibly discuss that every classroom is characterized by heterogeneity in terms of student ability level, readiness/prior knowledge level, and learning pace. Therefore, the teacher needs to find a way to help students work with the content at an appropriate level of challenge. One way to do this is by **using 'Enablers' and 'Extenders' which are at a level that builds on students' prior knowledge and prompts continued learning** (as such, these tasks correspond to the Vygotsian idea of working with students at their zone of proximal development). Tiered activities make it easier for the teacher to handle task complexity because they:

- Help all students focus on <u>key</u>/<u>core</u> concepts and ideas
- Allow students to begin learning from where they currently are
- Adjust the task by complexity, number of steps, and independence to ensure that students work at appropriately challenging tasks (not too hard, not too easy)
- Extend concepts and principles based on student readiness
- Allow students organize their work according to their learning style and use a variety of resources/materials at different levels of complexity
- Allow all students to participate and learn (e.g., students who are bored or find the core task difficult may interrupt the lesson)
- Enable students who need more support and guidance with the 'Core task' and extend the thinking of those students who succeeded in solving the task quite quickly and accurately

Then, teachers are provided with a text which uses a specific metaphor (i.e., *'The Ladder of Task Differentiation)* to help them understand the role of Enablers and Extenders (i.e., Tiered Activities) as two useful differentiation tools. Allow teachers 5-6 minutes to read the text and initiate a discussion by asking teachers to describe and explain the representation on page 35 and what they understood from reading the text. You can have the representation on a power point slide to help the discussion run smoothly and emphasize the meaning behind this metaphor: The extended task helps a teacher to differentiate the Basic/Core task <u>at least</u> a level up; some students might be ready to work with the Basic/Core task; while others need to work on a modified version of the Basic/Core task, <u>at least</u> a level down, because they are lacking necessary pre-requisite knowledge. Help teachers through this discussion realize that the ladder might consist of more than one extender and one enabler steps, depending on the composition of the class, the difficulty of the task, students' interests, readiness levels, etc. In fact, each step in the ladder might correspond to a different group of students, depending on the students' readiness levels, interests, prior-knowledge, and difficulties. During this discussion it could also be emphasized that the simplified version of the ladder (with three steps) can serve as a starting point to manage complexity. Once teachers are comfortable with working with tasks at three levels, they can work with incorporating more steps/modified tasks in their teaching.

In the next part of this activity, teachers are given three mathematical tasks[2] as 'Basic/Core tasks' and are asked to generate a tiered activity which can have either one or more readiness levels/levels up and down), so that all students participate and learn. In the interest of time and according to the needs of your audience, you can leave participants to work on one or more of these tasks (e.g., you might prefer to skip the pre-primary task if the participants are upper primary teachers). The goal is for each teacher to suggest <u>at least</u> one Enabler and <u>at least</u> one Extender for at least one of the tasks in order to apply what they have learned from the previous activity parts. Allow teachers approximately 10 minutes to work on this task individually or in pairs. Then, if there is available time initiate a discussion in which teachers present their ideas/suggestions (not in a written form). You can use a flipchart or a power point slide to write down their ideas and share these ideas with them via email after today's session. Each time ask them to interpret for other participants how they thought about the varying "degrees of difficulty" in the different tiers using differentiation language (e.g., more structured/open tasks, offer less/greater independence, for students with slower/quicker learning pace, simplier/more complex task/question, degrees of difficulty/readiness, interesting, engaging, focuses on key/essential ideas, for advanced/struggling learners, etc.) to do so. If there is not sufficient time to do that, some of this discussion could be incorporated in the Closing activity.

Possible enablers and extenders for **Task 1** (Core Task's Level of Challenge: Doing-mathematics):

Possible Enablers	Possible Extenders

[2] Participants may consider that the current/suggested tasks are not close to the tasks they use in their daily practice and they may face difficulties in thinking and developing enablers and extenders for them. You may provide tasks that participants bring or use in their lessons and develop extenders and enablers for them. Alternatively, the discussion can focus on one of the given tasks (e.g., the second one) and work as a team for identifying its cognitively demanding parts and suggestions for differentiating the task.

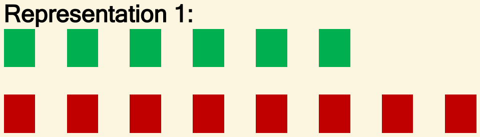
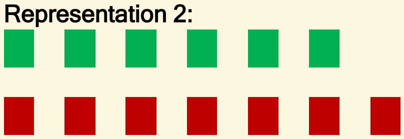
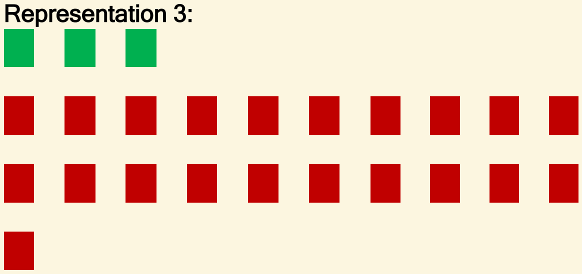
1. For students who have difficulties with identifying/ sequencing numbers: You have been given two special dice - one red and one black. Look at the red dice. What is the smallest number on it? What is the largest number? Can you say a number that is between 1 and 6?	1. Encourage students who are comfortable with questions 1 to 4 to move to question 5 as early as possible and then work with one of the following extenders.
2. Students who find adding difficult can spend a lot of time on questions 2 and 3. They could be asked to order their answers for questions 2 and 3.	2. Students who successfully complete question 5 will be given the following bonus task: ***Play Dice Bingo or 'Three in a Row' using two dice*** *(Children can use each dice separately e.g., if they throw 1 and 3 they can either cover 1 and 3 or simply 4.)* 4 | 5 | 7 9 | 2 | 10 3 | 6 | 8
3. Students who are struggling with question 5 could be asked, "What would happen if you kept one die the same and rolled only the other die?"	3. The commutative property may be raised by some students, who can be asked to explore it in this lesson.
4. In all cases - correct and incorrect - students will be asked how they are sure their answer is correct and/or to describe the strategy they used to get their answer.	4. For students who finish with the given task early: We also have dice that have ten faces, from 1 to 10. How would you find out all the possible sums you could make with two of those dice?
5. Students will be asked to record the sums in a format they find easy to handle.	5. For students who finish with the given task early: (a) How do you know that you have figured all the possible sums you can have with two dice? (b) How would you find out all the possible sums you could make with three dice?

Possible enablers and extenders for **Task 2** (Core Task's Level of Challenge: Doing-mathematics):

Possible Enablers	Possible Extenders
1. Some students may think that the second surface is larger because larger shapes were used; they might think that the first surface is bigger because more shapes were used. Indicative questions for students with misconceptions: - Check your assumption. How did you determine that this surface is bigger than the other one? Think of a way to prove it to me. - How big is surface A? How big is surface B? How bigger is surface A than surface B? - The size of surface A is 6 triangles. The size of surface B is 3 rhombi. Which is bigger? Can I tell? How?	1. For students who find a way to compare the two shapes, the teacher can ask them to think of another/alternative way to compare the surfaces.
2. For students who are struggling with identifying the shapes or seeing relationships between the shapes/ surfaces: - They could be provided with them two simpler surfaces to compare, and identifying different ways of comparing them (e.g., cutting them, putting the one over the other, etc). - The teacher could give them different pattern blocks and ask them to identify relationships betweent them: how many times does a triangle fit into a rhombus? A hexagon? etc.	2. For students who finish with the given task early: They could be asked to compare the size of the green and blue surfaces with a third surface made of two red trapezoids or with one yellow hexagon. - Which of those surfaces is the biggest/smallest? - Are they equal? - Sort the surfaces based on their size.
3. Some children may have difficulty getting started or thinking about a solution. Indicative questions for students who do not know how to get started:	

- With which of the two kids do you agree? Why? - Tell me in your own words what you understood you should do. - What makes it difficult for you to compare the two surfaces? -You have these materials at your disposal (e.g., pattern blocks, scissors etc.), how can they help you compare the surfaces? - When you had two papers and wanted to see if one was bigger than the other, what would you do? How can this help you compare these two surfaces?	

Possible enablers and extenders for **Task 3** (Core Task's Level of Challenge: Procedures-with-connections):

Possible Enablers	Possible Extenders
1. <u>For students who struggle to understand that the difference in the ratio actually corresponds to the money that Constantinos was left with:</u> - They could be provided with a simplified version of the problem in which Constantinos had 2 euros left over. - Provide them with two or three different representations and ask them to match the problem with the suitable representation. **Representation 1:**  **Representation 2:**  **Representation 3:**  • Match the problem with the suitable representation. Why do you think that this representation is more suitable than the others? • How are the amount of the 21 euros and the money of each child illustrated in this representation? • How can the representation you selected be used for solving the problem?	1. <u>For early finishers:</u> - Ask them to think of another way to solve the problem. - Ask them to pose a problem that follows the same logic as the one given.

2. <u>For students who focus on additive instead of multiplicative comparisons</u>:
   - Suggest a representation which illustrates the relationship between the money of the two children. How is the amount of 21 euros represented?
   - Suggest some ratios equivalent to 3:4.
   - Mary suggested that if Andreas had 9 euros, Constantinos shoud have 10 euros. In constrast, Tina suggested that if Andreas had 9 euros, Constantinos should have12 euros. Do you agree with Mary or Tina? Justify your answer.

2. <u>For early finishers</u>:
   - Change the problem by adding a third child into the story e.g.: Andreas, Constantinos, and Marcos had some savings with a ratio 3:4:5 respectively. They decided to buy a birthday present for their mother sharing the cost equally. After they bought the present, Andreas had spent all of his money. Marco had 16 euros left over. Find the price of the present as well as how much money each of the two brothers spent to buy it.

3. <u>Students who lack covariational thinking:</u>
   - Mona started a table that shows the amount of money of Andreas and Constantinos. Decide what should go into the blank cells of the table.

Andreas's money	Constantinos's money	Difference between the amounts of money
3	4	
9		3
	20	
21		
	100	
		3
		21

4. <u>For early finishers</u>:
   - How much would the present cost if Constantinos spent twice as much money as Andreas spent and he had 12 euros left over?

After working on developing enablers and extenders for the given task(s) of this activity, teachers can read the principles of modifying tasks on page 37 and think individually for 2 minutes whether their modified versions of the task(s) take into account these differentiation principles. Then you can raise a whole-group discussion to share their ideas/responses and explain these principles. You can share the diagram found in Appendix 5 with the teachers which can help them in developing tiered activities in the future.

Connections to My Practice

Indicative Duration: 10 minutes

In this activity, teachers are asked to videotape a lesson in which they will teach one mathematically challenging task and employ 2-3 differentiation practices discussed in today's meeting (the Closing Activity will help the teachers codify these practices). For some of the logistics involved in videotaping you can read the detailed instructions in *Connections to My Practice* in Case of Practice 1. Ensure that teachers understand that they need to differentiate the task up and down to help all students work productively on the task. Finally, teachers should select and send you the selected videoclips that are illustrative of their attempts to differentiate their approach regardless of how successful these attempts were. The focus of this particular activity is to (a) watch their lesson, (b) notice how they have implemented some differentiation practices as discussed in today's session with their students, (c) consider whether these practices helped all students being productively engaged with the task, and also (d) identify possible problems they have encountered during the task enactments. Remember to provide directions as to what needs to be videotaped, how the setting of the videotaping is going to be, and what the focal points are (the guiding questions can help teachers understand the focus of the activity).

Closing Activity

Indicative Duration: 5 minutes

This is a short activity for helping teachers codify the differentiation practices discussed in today's meeting. Allow teachers 4-5 minutes to work in pairs and name some differentiation strategies they have considered for adjusting the level of mathematical challenge of a given task. You can codify the differentiation practices in a word document and share it with the teachers via email after today's session. Teachers could refer to the following differentiation strategies:

- Developing tiered activities for at least three student levels (introductory level; those at the standard level; and those who are capable of more in-depth higher-order tasks)
  - Enablers: for students (not necessarily the same each time) who need some kind of support to 'enable' them work on the core task
  - Extenders: for students (not necessarily the same each time) who can go beyond the core task
- Modifying the task complexity (either up or down) by using one or combinations of the following approaches:
  - Adding or relaxing task constraints
  - Changing the conditions of the problem
  - Changing the numbers of the problem
  - Asking for generalizations and pattern noticing
  - Asking for another/different solution
  - Inserting obstacles to the solution
  - Limiting the problem information provided/ representations
  - Decontextualising from specific cases
  - Adding questions such as:
    - What if …? Could it be possible?
    - Why?
  - How many solutions exist? How do we know that we have found them all?

Key Take-away Points of the Case of Practice 2

- A teacher could turn a mathematically challenging task into a mathematically non-challenging task or vice versa. As discussed in Activity 2, there are several factors that could influence this, such as the teacher's knowledge of content and teaching mathematics; students' readiness and prior knowledge; guidelines given from school inspectors of the subject specialists on what and how to teach it; pressure to cover the curriculum etc.
- The Ladder of Differentiation: One way to keep ***all*** students focused on essential understandings but at different levels of complexity, abstractness, and open-endedness so that each student is appropriately challenged pertains to developing and using **Tiered**

activities (i.e., enablers and extenders). **Enablers** offer extra support and guidance with the core task to students, while **Extenders** provide greater mathematical challenge than this provided by the core task to students who already solved the core task.

- While developing enablers and extenders, bear in mind that:
    - A student might be clustered in the first group for one task and in the second, for another.
    - Tasks should focus on learning objectives and essential concepts.
    - Tasks ought to respond to the specific learning needs of different groups according to ability, readiness, degree of support required and learning preferences.
    - All tasks should be engaging, active and interesting
    - Extender tasks should not be just "more work" and enablers should not represent "dumbed down" versions of the core task.