

International Conference of the European project* “Promoting Formative Assessment: From Theory to Policy and Practice”



Developing Educational Policies to Promote Formative Assessment:
The Contribution of Educational Research

THREE SECONDARY TEACHERS EXPLORE STUDENTS' UNDERSTANDING OF MATHEMATICS THROUGH FORMATIVE ASSESSMENT

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GREECE

We know that learning is multidimensional, therefore it cannot be adequately measured by a single technique.



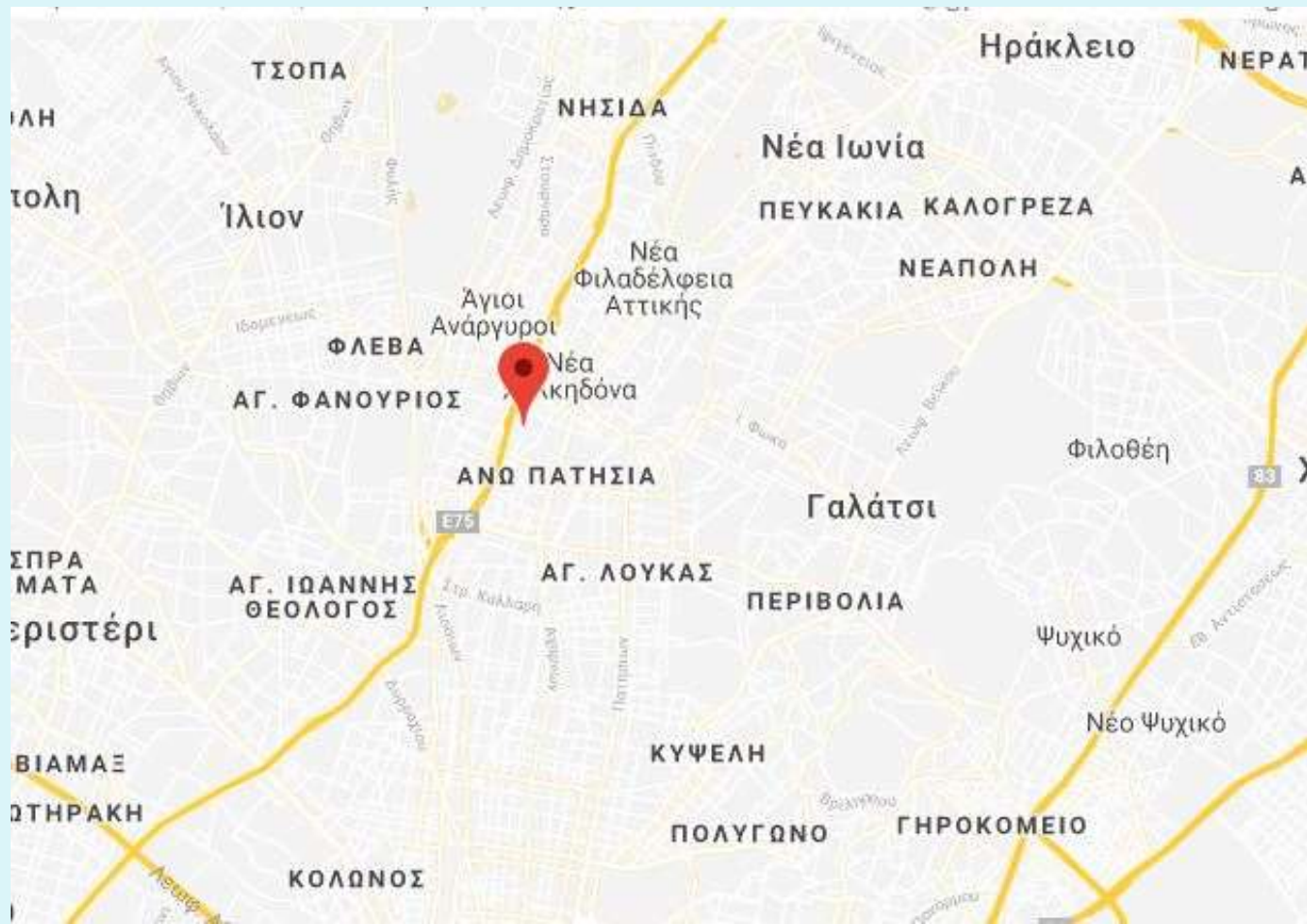
Using a variety of assessment techniques we give students the opportunity to show what they know and what they can do.

The choice of an assessment technique depends on the learning objective to be assessed

Let us introduce ourselves

We are three high school teachers who, through our participation in the Formas program, utilized various techniques for assessing the involvement of students with mathematics, with the aim of enhancing the learning and teaching process.

Indicatively and for the sake of variety, we have chosen to present you a different way of evaluation.



High School of Nea Chalkidona

Eleni Tsoukala

EVALUATING THE EXECUTION OF AN ACTIVITY

observing and measuring of
students' skills

Evaluating the execution of an activity:

It Includes execution of a set of activities

The role of the teacher:

The assessor monitors the process,
encouraging the students
themselves

- to assess the group's ability
- to perform the activity
- to extract the result.



- the modest teacher speaks.
- the good teacher explains
- the excellent teacher shows
- the great teacher inspires.

Description of activity:
Grade 8, trigonometry:
“tangent to an angle”.
Total students: 20,
5 groups of 4 people

Students were asked to
calculate the height of the
room from the edge of the
ceiling to the floor.



Learning objectives to be evaluated:



- Ability to design the solution steps.



- Ability to execute these steps.



- Ability to identify difficulties and finding a solution to them.

CRITERIA OF SUCCESS:

Ability to perform steps

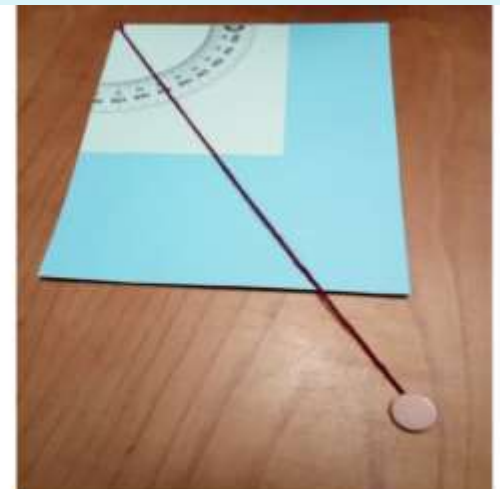
Data processing

The coordination of the group members in making a plan for the requested calculation.

Evaluation of results

Procedure:

Each group was given: **Measuring rule, goniometer, paper, pencil and pocket calculator.**



Each member of the group took on a specific role.

The first: Organized the mathematical modeling

The second: Took measurements

The third: Recorded the measurements

The fourth: Carried out the calculations and extracted the result.

In case someone encountered any form of difficulty, he could ask for help from the other members of the group.



At the end, the results were presented to the plenary session and the teacher asked the students

A. to comment on the different results found by each group

B. To evaluate the activity.

The students, after discussing this activity within their group, concluded that

they are able to calculate the height of any inaccessible point, as long as they apply the same process.

Techniques used to assess the ability to achieve their objectives:

Involvement of students in the assessment process

The teacher, together with each group of students, discussed and evaluated every step of the process. The team noticed the errors and redefined the next steps.

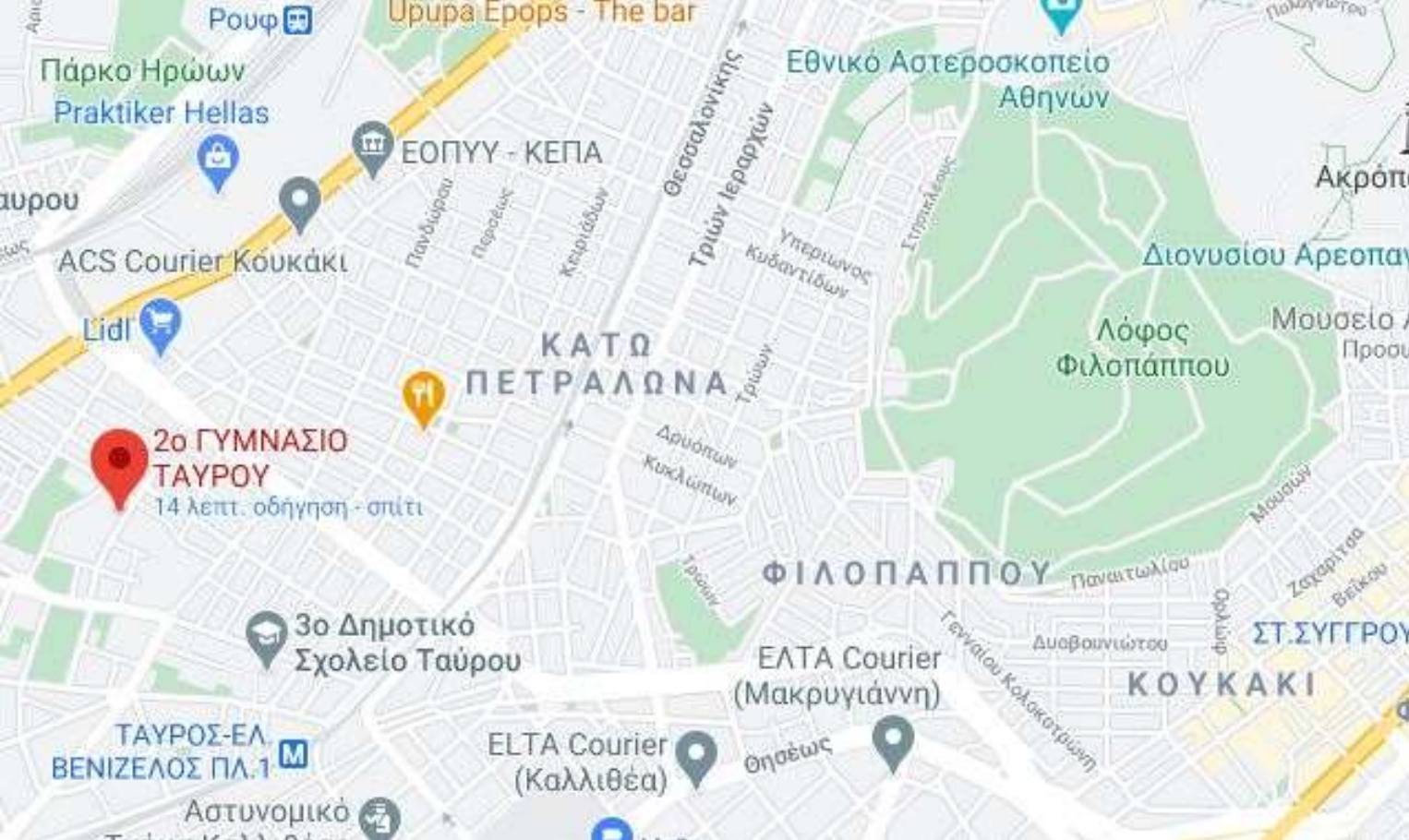
In this way, each student evaluated himself (self-evaluation) as well as the other members of the team (hetero-evaluation).

Evaluation of the students' experience upon completion of their participation in the assessment process

The students felt that their involvement in the assessment of their skills, helped them to identify their weaknesses and the points they need to strengthen

But most of all, they think that it helps

in creating a positive learning culture.

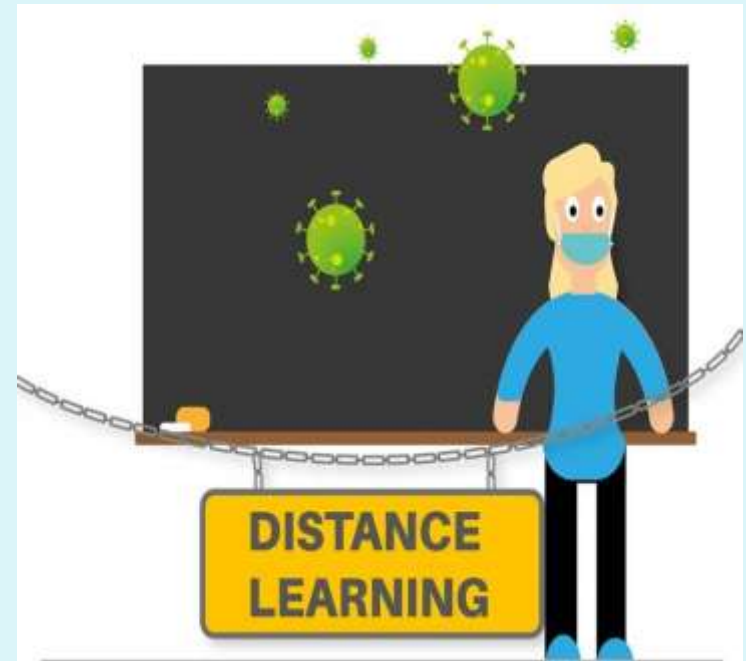


2nd Gymnasium of Tavros

Ardavani Kalliopi

Self-assessment - Hetero-assessment

The 9th grade students had been taught the rational equations in the classroom. With the lock down of school due to covid-19 epidemic, there is an opportunity for repetition and evaluation of the students, as teaching is undertaken remotely and on digital platforms.



So, the first week in the online class, the students are asked to put in order the steps for solving the equation

and then... to match these steps with the parts of a solved equation.

These activities were made with the Hot Potatoes software and students engaged in self-assessment.



the next week in the online class:

students are asked to write the prerequisites for the rational equation solution and then to study 4 solved exercises, to point out mistakes, to assess and to grade them.

12 9th grade students participated.

Homework is done remotely by students at home.

Rearranging the steps to solving the rational equation

solving a rational equation

guide

έλεγχος

Αναίρεση

Ξανά

clearing the fractions

multiplying both sides of an equation by the same quantity (the LCD)

factoring all the denominators

find the LCD of every term

check for extraneous solutions

finding excluded values of rational functions

with the distributive property we do the multiplications

to solve a rational equation:

solve the non rational equation

And then they map these steps to the parts of a solved rational equation.

Λύση κλασματικής εξίσωσης εφαρμογή

Ταίριαξε τα αντικείμενα στα αριστερά με τα αντικείμενα στα δεξιά

Check

Να λυθεί η εξίσωση:

$$\frac{2x}{x^2-9} - \frac{1}{x-3} = 1$$

Αναλύουμε τους παρονομαστές σε γινόμενο παραγόντων

Με τη μηχανή όσους κάνουμε τους παρονομαστές

εύκολα = 3

Προσδιορίζουμε τις τιμές του αγνώστου για τις οποίες όλοι οι παρονομαστές είναι διάφοροι του μηδενός

Κάνουμε απλοκή παραγόντων σε αριθμητή & ονομαστή

$x \neq 3$ & $x \neq -3$

Βρίσκουμε το ΕΚΠ των παρονομαστών

Επιλέγουμε τη μέγιστη που ταιριάζει

$$(x-3)(x+3) \left[\frac{2x}{(x-3)(x+3)} - \frac{1}{x-3} \right] = (x-3)(x+3)$$

Η λύση της εξίσωσης είναι $x=-2$

Πολλαπλασιάζουμε και τα δύο μέλη της εξίσωσης με το ΕΚΠ των παρονομαστών

Από τις λύσεις που βρήκαμε απορρίπτουμε εκείνες που δεν ικανοποιούν τους περιορισμούς

$$2x - (x-3) = x^2 - 9$$

$$(x-3)(x+3) \left[\frac{2x}{(x-3)(x+3)} - \frac{1}{x-3} \right] = (x-3)(x+3)$$

$$\frac{2x}{(x-3)(x+3)} - \frac{1}{x-3} = 1$$

$x=3$ απορρίπτεται, $x=-2$ δεκτή

$$\text{ΕΚΠ} = (x-3)(x+3)$$

Task 1

Rational Equations

Description:

In our theory we saw a compass-guide for the solution of rational equations.

Is that enough?

Should the student in each step of the guide have more knowledge than previous lessons - modules?

TASK 1

Determine the most knowledge one must have to solve a rational equation (count) at each step of the guide

e.g. 1: To factor the denominators one must know the standard identities as well as ...

Students said:

Almost everyone described the steps to follow for solving the rational equation and many pointed out - identified secondary knowledge. (prerequisites)

Specifically:

One must know :

Standard Identities : 9

How to factor : 8

How to find LCD: 6

The Distributive property: 10

Priority of operations: 3

Combining like terms: 4

Solve first and second degree equations: 9

the quadratic formula :3

Task 2 – Correcting solved exercises with errors

Μαθητής Α

$$\frac{2x}{x^2-9} - \frac{1}{x-3} = 1$$

$$\frac{2x}{(x+3)(x-3)} - \frac{1}{x-3} = 1$$

$$\frac{2x}{(x+3)(x-3)} - \frac{1}{(x+3)(x-3)} = 1$$

$$2x - (x+3) = 1$$

$$2x - x - 3 = 1$$

$$x - 3 = 1$$

$$x = 4 \quad \Delta \text{ λύση}$$

Πρέπει $x \neq 3$ και $x \neq -3$

Μαθητής Β

$$\frac{2x}{x^2-9} - \frac{1}{x-3} = 1$$

$$\frac{2x}{(x+3)(x-3)} - \frac{1}{x-3} = 1$$

$$\frac{2x}{(x+3)(x-3)} - \frac{1}{(x+3)(x-3)} = 1$$

$$2x - x + 3 = x^2 - 9$$

$$x + 3 - x^2 + 9 = 0$$

$$-x^2 + x + 12 = 0$$

$$x^2 - x - 12 = 0$$

$$\Delta = b^2 - 4ac = (-1)^2 - 4(-12) = 1 + 48 = 49 > 0$$

$$x_1, x_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{1 \pm 7}{2} = \begin{cases} 4 \text{ λύση} \\ -3 \text{ απαγορεύεται} \end{cases}$$

Πρέπει $x \neq -3$ και $x \neq 3$

Μαθητής Γ

$$\frac{2x}{x^2-9} - \frac{1}{x-3} = 1$$

$$\frac{2x}{(x+3)(x-3)} - \frac{1}{x-3} = 1$$

$$\frac{2x}{(x+3)(x-3)} - \frac{1}{(x+3)(x-3)} = 1$$

$$2x - (x+3) = x+3$$

$$2x - x - 3 = x+3$$

$$2x - x - x = 3+3$$

$$0x = 6$$

αδύνατο

Μαθητής Δ

$$\frac{2x}{x^2-9} - \frac{1}{x-3} = 1$$

$$\frac{2x}{(x+3)(x-3)} - \frac{1}{x-3} = 1$$

$$\frac{2x}{(x+3)(x-3)} - \frac{1(x+3)}{(x+3)(x-3)} = 1$$

$$2x - 1(x+3) = 1$$

$$2x - x + 3 = 1$$

$$x = -2$$

- Study each student's attempt to find solutions to the rational equation.
- Find his mistakes
- Indicate exactly what he did wrong and why
- Evaluate whether these affect finding solutions
- Give a grade to each student's effort and justify it

The grades assigned

11,12,13,13,13,14,14,**15**,15,20

$\frac{2x}{x^2-9} - \frac{1}{x-3} = 1$ (Μαθητής Α)

$\frac{2x}{(x+3)(x-3)} - \frac{1}{x-3} = 1$

$(x+3)(x-3) \cdot \frac{2x}{(x+3)(x-3)} - (x+3)(x-3) \cdot \frac{1}{x-3} = 1$

$2x - (x+3) = 1$

$2x - x - 3 = 1$

$x - 3 = 1$

$x = 4$ Δεσφά

Πρέπει $x \neq 3$ και $x \neq -3$

Student Th. makes a detailed grading (unique):

For St A : total score 15 out of 20 as he correctly factored all the denominators **4/4** Identified the restrictions **2/2** determined the LCD but he didn't apply it correctly on both sides **2/3**, he cleared correctly the fractions on the terms he applied the LCD **1/2**, he did the multiplication correctly on the terms he applied the LCD **2/3**. Finally, he solved correctly the regular equation, but found the wrong result **4/6**

For Student A: So this student has made **one mistake** in not multiplying all the terms of the equation with the LCD. As a result, he did not put $(x + 3)(x - 3)$ next to 1 and **this cost him continuously and in the result** But because he has put it in all the other terms, I might take it as **a mistake of carelessness**. I would assign him **14/20**.

The grades assigned :

15, 16, 16, 17, 18, 18, 20, 20, 20, 20

For St B' :

In the 4th line he opened the parenthesis without distributing the negative sign. He made this mistake **did not know that if there is a minus before a parenthesis, we must change the sign of the terms in parentheses when we open it.** It is an serious mistake

because in the process he did wrong operations. **Total score: 15**, because he knows the process of solving a rational equation but overlooked the minus sign in front of the parenthesis.

The image shows a student's handwritten work on a piece of paper. At the top, the name 'Matheus B' is circled in blue. The work shows the following steps:

- Line 1: $\frac{2x}{x-9} - \frac{1}{x+3} = 1$
- Line 2: $\frac{2x}{(x+3)(x-3)} - \frac{1}{x+3} = 1$
- Line 3: $\frac{2x}{(x+3)(x-3)} - \frac{1}{x+3} = 1$ (with a minus sign written above the fraction)
- Line 4: $\frac{2x}{(x+3)(x-3)} - (x+3)(x-3) = (x+3)(x-3)$ (with a minus sign written above the term)
- Line 5: $\frac{2x}{(x+3)(x-3)} - x + 3 = x^2 - 9$ (the term $-x + 3$ is circled in red)
- Line 6: $x + 3 - x^2 + 9 = 0$
- Line 7: $-x^2 + x + 12 = 0$
- Line 8: $x^2 - x - 12 = 0$
- Line 9: $\Delta = b^2 - 4ac = (-1)^2 - 4(-12) = 1 + 48 = 49 > 0$
- Line 10: $x_1, x_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{1 \pm 7}{2} = \begin{cases} 4 \\ -3 \end{cases}$
- Line 11: 'Then $x \neq -3$ and $x \neq 3$ '

For St B' :

St B **has forgotten** to put parentheses in the operation $x + 3$. The error results in finding the wrong signs which will lead to the wrong solution.

Total score for student B is **17** since he applied all the other steps correctly and the **mistake** he made was purely **careless** and he could solve the equation correctly.

The grades assigned :

6, 12, 13, 13, 13, 14, 14, 15, 16, 16

For St C:

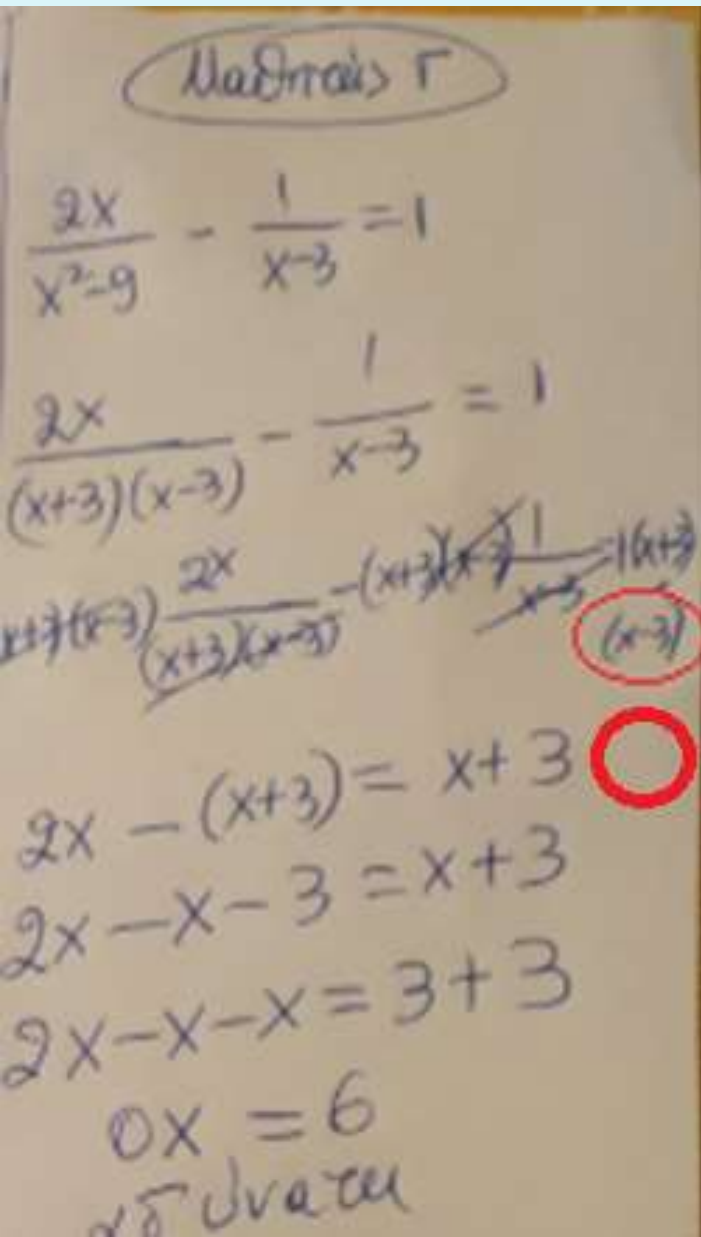
This student has factorized, he has found the LCD all OK. But then when he went to clear the fractions and write a normal equation while he has written correctly $2x - (x + 3) = x + 3$, he forgot to put $(x-3)$ next to $x + 3$.

He has probably just forgotten it or is blind.

The equation should result in:

$2x - (x+3) = (x+3)(x-3)$. I do not know these errors are annoying because they are simple a matter of copying.

I would give him a 13.



The grades assigned 3,6,8,10,**10**,11,11,11,13,15

QUESTION Δ

$$\frac{2x}{x^2-9} - \frac{1}{x-3} = 1$$
$$\frac{1}{2x} - \frac{1}{x-3} = 1 \quad !$$
$$\frac{2x}{(x+3)(x-3)} - \frac{1(x+3)}{(x+3)(x-3)} = 1$$
$$2x - 1(x+3) = 1 \quad ?$$
$$2x - x + 3 = 1$$
$$x = -2$$

For Student D :

He made a mistake in the 4th line because where he has cleared the fractions, but he does not multiply with the LCD, the 2nd side. He may have made this mistake **out of sheer momentum or carelessness.**

It affected the solution

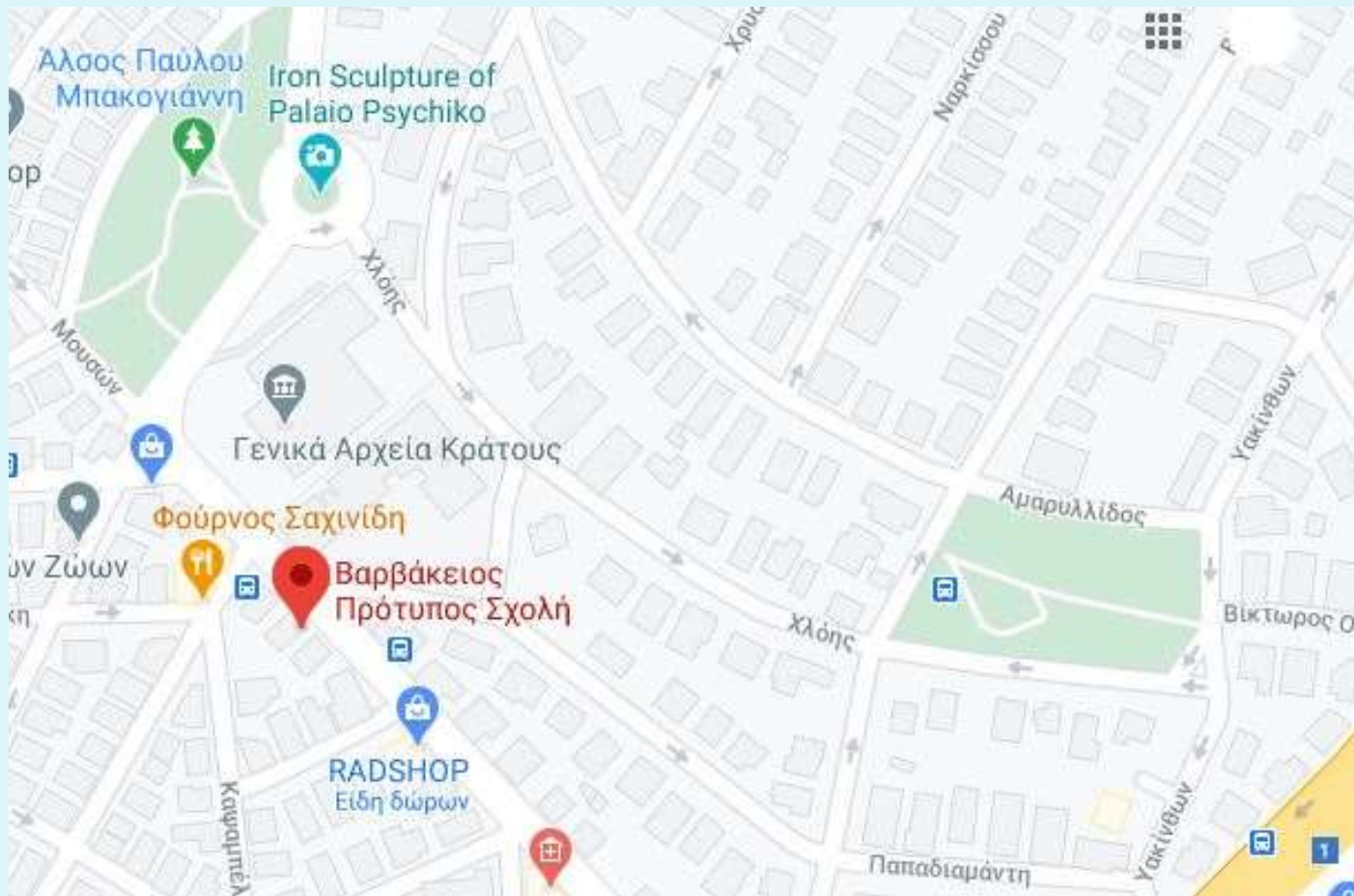
because he did not do the correct operations on the 2nd side. **Total score 10** , since he forgot the main step in the solution process, that is multiplying both sides with the LCD.

Valuation

The benefit of engaging in this activity is that the students:

- were forced to think - realize the **prerequisites** and consequently each of them his weaknesses (on a personal basis) for the steps of solving the equation,
- focused on the **standard mistakes** that happen when solving an equation
- assessed the **severity of mistakes** through the need to document their grade
- **Evaluated** numerically the error with the scoring performance

They entered a different role: the teacher-assessor



Varvakeio Model Junior High School

Eirini Kouletsi

Self-Assessment/evaluation

As a tool for Formative
Evaluation

Self-assessment: Involving students in the assessment process

Its importance is **crucial** for the "autonomy" of the student, who through self-assessment:

- Has the ability to think about learning issues and thus enhance the development of metacognitive skills
- Has several opportunities to focus on specific areas. He is able to understand the way of evaluation (and through the rating), does not consider the points arbitrary nor does he try to improve his performance solely by this criterion
- Contributes to the improvement of his image while developing his responsibility and self-knowledge
- Contributes to its better feedback.



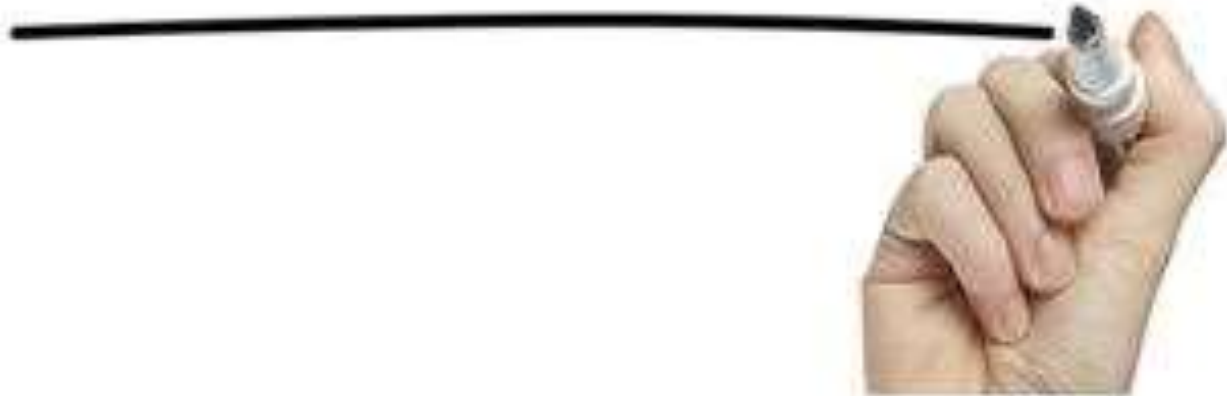
Roles

The student: at the center of the process.

The teacher:

- In the background, monitors the process
- Helpful, when the student has difficulty assessing the test
- encouraging the students to evaluate the test.

ACTION PLAN



CLASS 1

On October 14, 2012, Austrian Felix Baumgartner became the first person to break the sound barrier by falling from the stratosphere. The whole project cost € 23,000,000. Felix's uniform was 0.6% of the total amount, the balloon increased by 15,000 €, the balloon 0.9% of the total amount, the sun for the balloon cost 1/3 of the value of the uniform.

- How much did the uniform, the balloon and the sun cost?
- The remaining amount was used for the technical support of the shipment and the payment of the technicians. The amount of technical support was four times that of the technicians. The 5 technicians shared the money equally. How much money did each of them get?

CLASS 2

percentages

PROPORTIONALIT

It is known that the flour produced, when wheat is ground, has a weight of 20% less than the weight of the wheat that is ground. When the flour is kneaded to become dough, it increases its weight by 50%. Finally, the dough when baked in bread loses 20% of its weight. We have 500 gr of wheat.

- a. Calculate the weight of the bread to be produced
- b. Calculate how many percent less will eventually be the weight of the bread than the original weight of the wheat.

b) $153.000 + 192.000 + 51.000 = 396.000$ } 2,5 points
 Remaining amount: $2500000 - 396000 = 22.604.000$

Let $x \in \mathbb{E}$ be the technicians' payment
 then $4x \in \mathbb{E}$ for technical support } 2 points

$4x + x = 22604000$ (mathematical model) }
 $5x = 22604000$ } 3,5 points
 $x = 22604000 : 5$
 $x = 4520800$

$4520800 : 5 = 904160 \text{ €}$ for each technician } 2 points
 Total : 10 points

MODEL-DISCUSSION

Take time
to reflect



Students' Comments Grade 8 (Class 1)

- **UNDERSTANDING TEACHER
EVALUATION CRITERIA:**

"It was very helpful because I understood how you evaluate "
"I understood the reasoning of the teachers when they correct,... their criteria",
"I understood the teacher's point"

8/24

Take time
to reflect



Students' Comments Grade 8 (Class 1)

• UNDERSTANDING ERRORS

"We identified our weaknesses to improve them" "The Self-Assessment was a nice experience, because I see my mistakes better and understand them faster", "It will prevent me from repeating such mistakes in the future". "It was enjoyable and I realized that I have to practice in the divisions" "I could see my mistakes better, because when I get the test corrected I give, wrongly of course, more importance to the grade and I do not see my mistakes globally"» [13/24](#)

Take time
to reflect



Students'
Comments
Grade 8
(Class 1)

"We managed to sharpen our critical ability" "Based on the correct answers I checked the test and I knew about my grade"

Students' Comments (Class 2)

Take time
to reflect



- **UNDERSTANDING TEACHER EVALUATION CRITERIA:**
 - "The experience was useful as it opened my eyes on how teachers grade our mistakes" 7/24
- **UNDERSTANDING ERRORS:** "We were able to see our mistakes on our own" "I think that when we evaluate ourselves it is easier to remember our mistakes and to limit them" 14/24

Students' Comments (Class 2)

Take time
to reflect



- **IMAGE OF ONESELF - DIFFICULTY OF (SELF)-EVALUATION:**

"We get a better picture of ourselves" "it's a bit difficult to evaluate yourself" "you have to judge yourself objectively", "the most difficult thing was as we saw our careless mistakes we could not correct them, but it was a very good way to judge ourselves objectively", "helps us have better self-criticism" "It is a way to become better judges"

MAKING GOOD USE OF OUR EXPERIENCE

- Assistance on learning and achieving the best possible learning outcomes
- Feedback on the course of teaching, on improving teaching and the overall operation of the school

But mainly in

PLEASANT EXPERIENCE

Creating a positive learning culture

"I would like to repeat it",

"I would like to adopt this way of evaluation in classroom",

"Apart from the fact that it reduces stress it gives you the opportunity to correct your mistakes, yourself and be honest with yourself “

BENEFITS FROM THE PARTICIPATION IN THE FORMAS PROJECT

- It is useful to evaluate each step in the educational process
- It is important to set the criteria on which the evaluation will be based
- Students' participation in the assessment process helps them understand their mistakes as well as the gravity of each mistake.

THANK YOU!