

# Learning and assessment in mathematics: Is it possible to integrate?

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# Summary

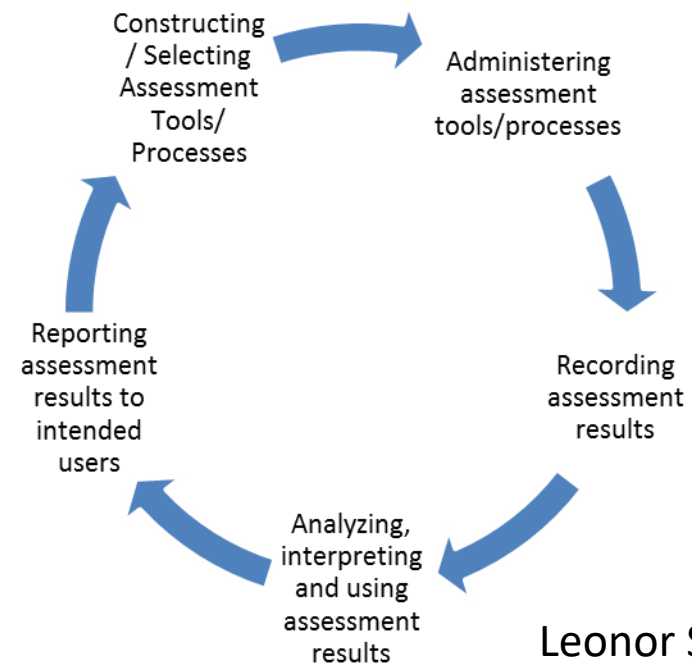
- The problem
- A complementary strategy to solve the problem
- Conclusions

The problem

# What means to assess?

To assess is a process, an **activity of communication**, that includes the collection of data about mathematics learning with **intentionality**, an interpretation of the data collected, and an action supported by this interpretation.

In accordance with the perspective of the project *Formas*



# What means to assess?

To assess is a process, an **activity of communication**, that includes the collection of data about mathematics learning with **intentionality**, an interpretation of the data collected, and an action supported by this interpretation.

To assess the student learning, in a certain moment, to inform, to select, ...

Summative  
assessment

For what?  
Which  
purposes?

To assess the student learning to improve learning and teaching

Formative  
assessment

# Conceptual clarification?

*Formative assessment and summative assessment are not different in their types, but in their purposes* (Santos, 2016)

*Using the terms “formative assessment” and “summative assessment” can give the impression that these are different kinds of assessment or are linked to different methods of gathering evidence. This is not the case, what matters is how the information is used* (Harlen, 2006)

# The problem

Formative assessment contributes to improving students' mathematics learning

**but**

It is not a reality in most of the mathematics' classrooms in Portugal and several other countries

(eg., EC, 2011, Fernandes & Gaspar, 2014, McGatha & Bush, 2013, Torrance & Pryor, 2001)

# Several reasons...

## Teachers

- Knowledge about assessment
- Number of students per class
- Lack of time
- Suspicious about informality

## Schools

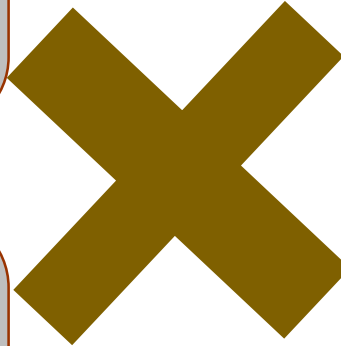
- Organization
- Not flexible curriculum
- Colleagues' pressure

## Society

- Parents' pressure
- Summative is a tradition
- Priority to knowledge

## Educational Policy

- External assessment (Mathematics)
- Accountability





# A complementary strategy...

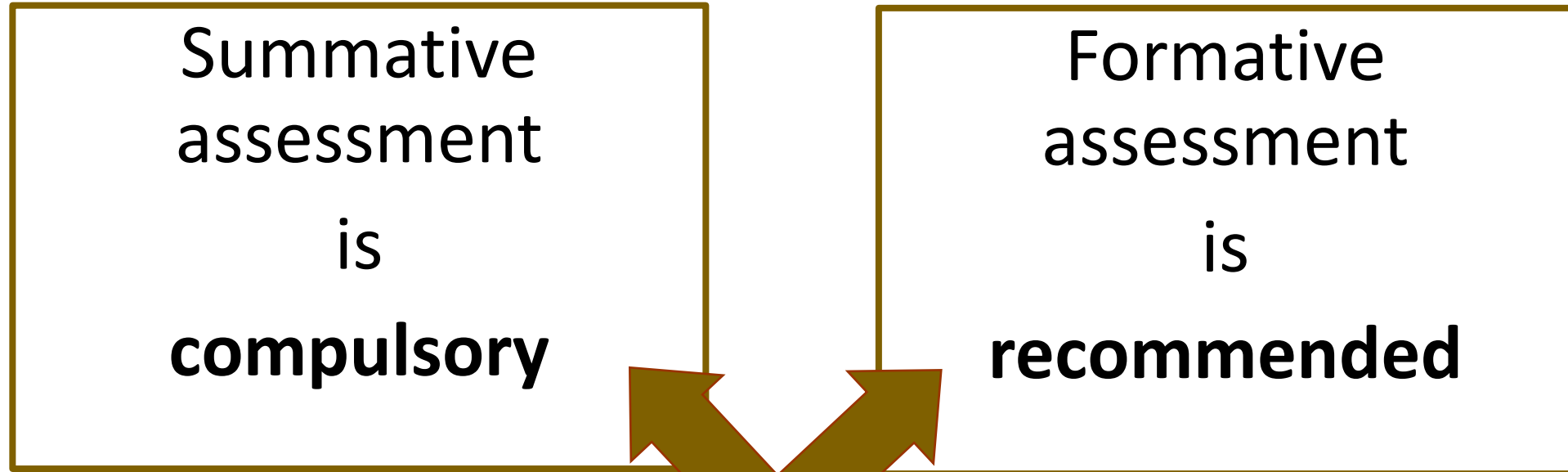
Summative  
assessment  
is  
**compulsory**

Formative  
assessment  
is  
**recommended**

*This complex relationship creates tensions in  
teachers*

(Bennett, 2011)

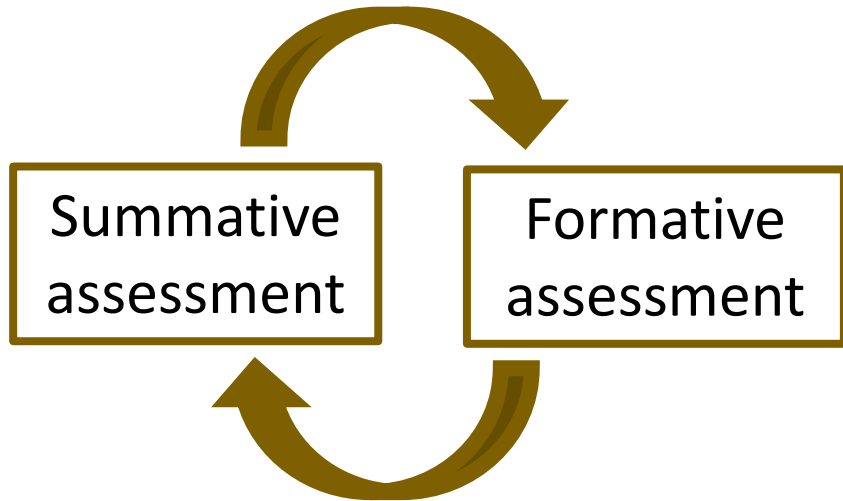
# A complementary strategy...



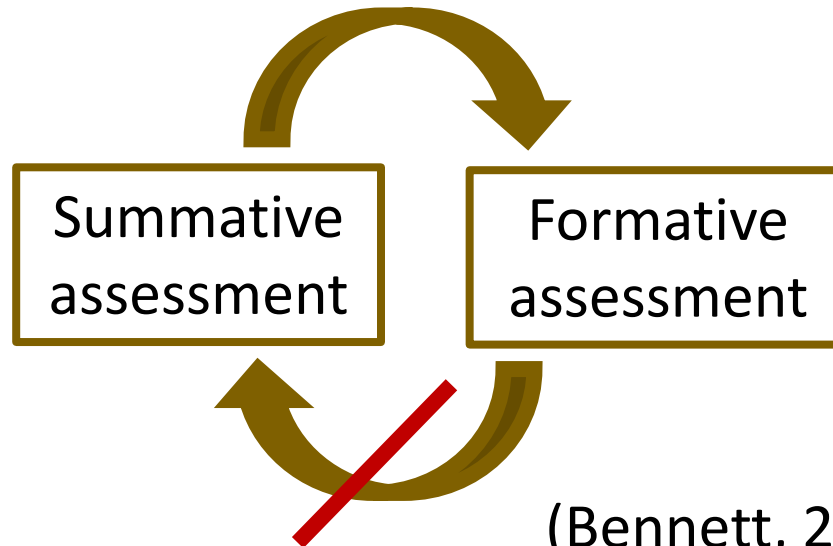
**Articulation?**

A complementary strategy  
to solve the problem

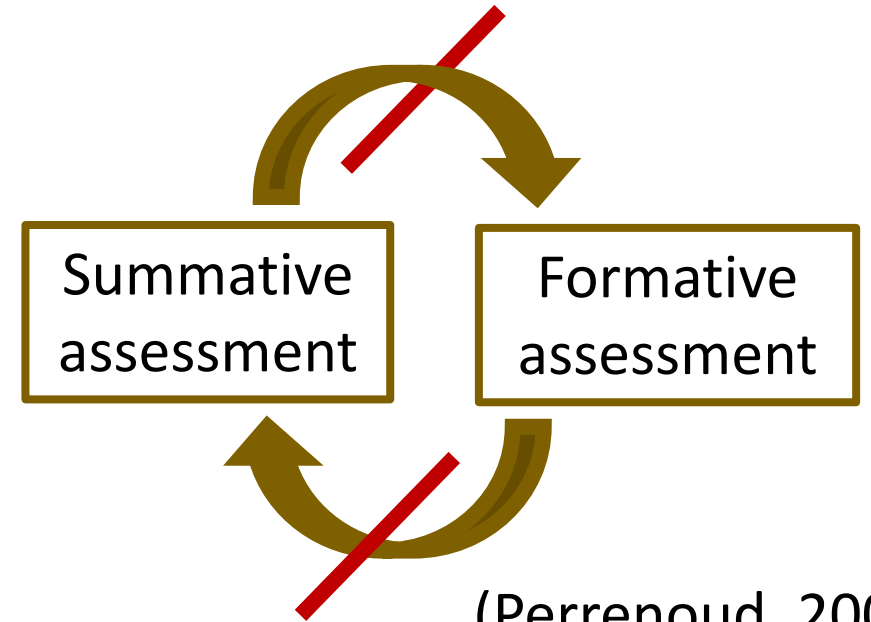
# Theoretical perspectives



(Black & Wiliam, 2003;  
Semana & Santos,  
2018; Shavelson *et al.*,  
2008)

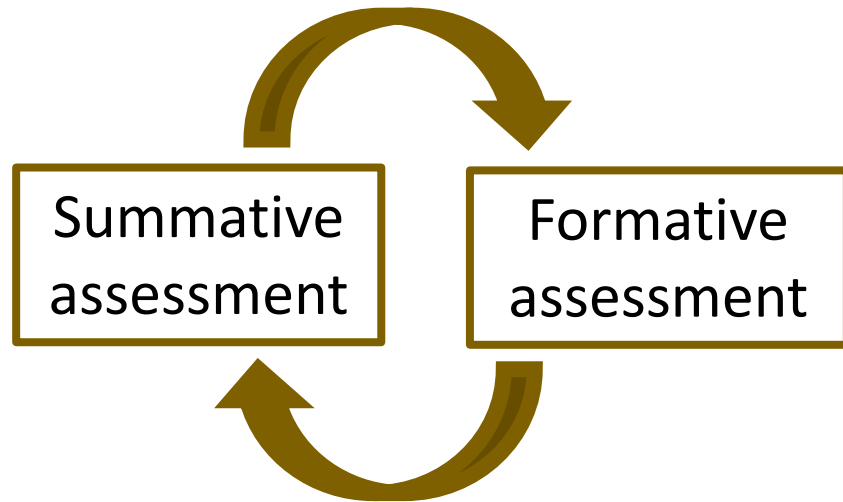


(Bennett, 2014)

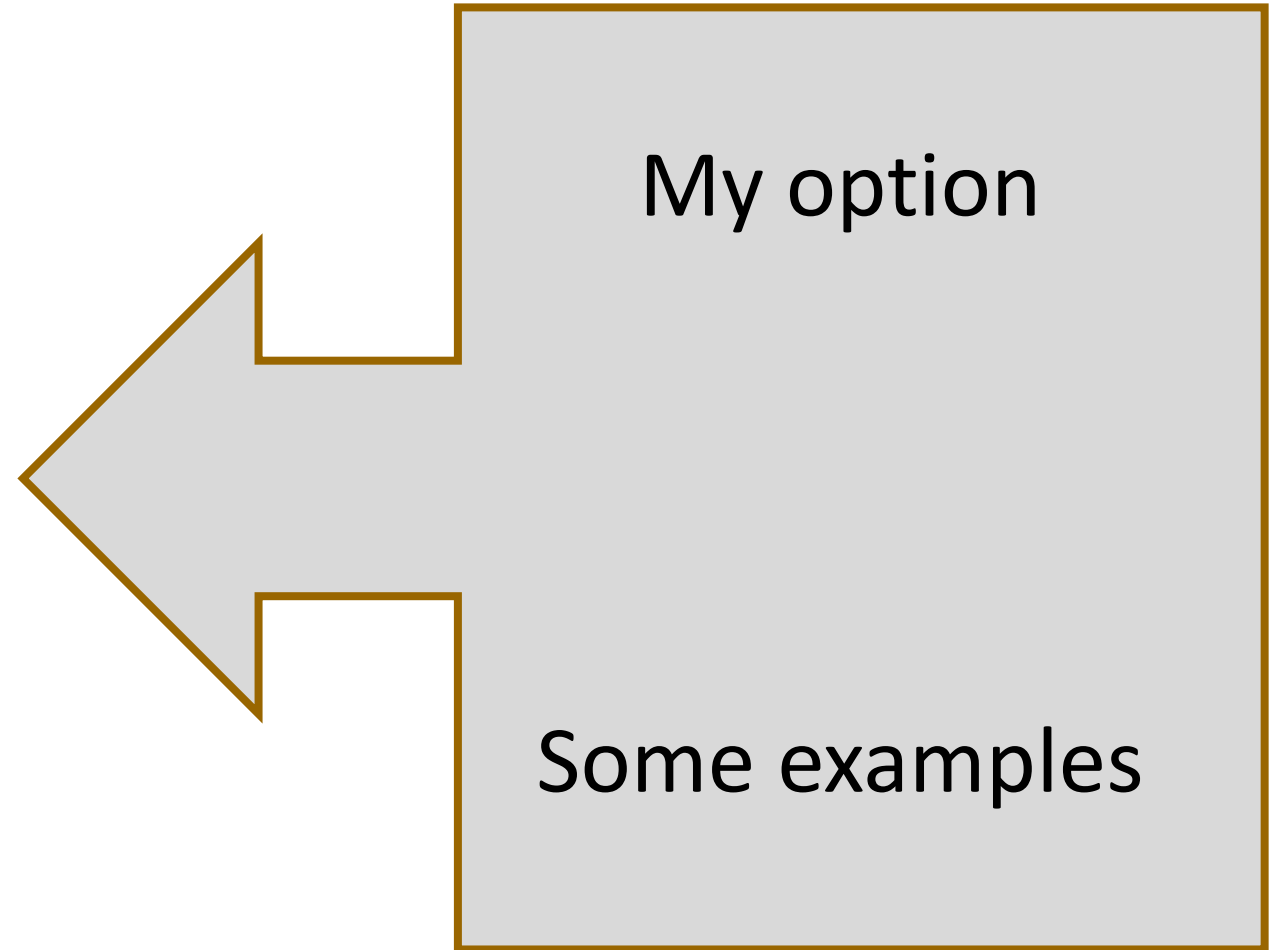


(Perrenoud, 2001;  
Shepard, 2001, Vial,  
2012)

# Theoretical perspectives



(Black & Wiliam, 2003;  
Harlen, 2006;  
Shavelson *et al.*, 2008)



# Study 1 – Test in two phases

(Santos, 1992)

Context: Project Mat<sub>789</sub>

Participants: Two classes of 8<sup>st</sup> grade (students with 13 years old)

Focus: Mathematics processes (mathematical reasoning, mathematics communication, problem solving)

Strategy: 1<sup>st</sup> phase in class with consultation; feedback; 2<sup>nd</sup> phase outside the classroom

Data collection: Students questionnaire, interviews to four students and teacher, classroom observation, documental analysis

## Results: Contributions for learning

- To reinforce the understanding about mathematics concepts
- To solve problems
- To communicate mathematically
- To develop responsibility
- To have self confidence in doing mathematics

## Contributions for a new culture of assessment

- Trust
- Stress reduction

# Teacher opinion

## **Formative intentionality**

*To reduce the stress that students usually have during a test (...) in addition to giving them more opportunities, it permits that they rethink the questions. It is perhaps an opportunity for them to have a different attitude towards mathematics and to get more confidence in themselves.*

**Final classification:** holistic scale, 1<sup>st</sup> + 2<sup>nd</sup> + evolution



## Study 2 – Learning portfolio

(Dias & Santos, 2016)

Context: Project A2PC2, Master

Participants: Three students of 11<sup>th</sup> grade (16 years old)

Focus: Different types of learning (chosen by the students)

Strategy: Monthly, different versions of the same task accompanied by explanations and justifications of the options made; feedback by email for each version except the last one; final reflection at the end of each school term

Data collection: Two interviews with each student, documental analysis

## Results: Contributions for learning

- Selection of the tasks: Orientation; organization; translation
- During the tasks: Reflection, self-regulation, development
- During the justifications: Communication (to agree, to question, to write)
- To develop responsibility, habits of work and persistency

## Contributions for a new culture of assessment

- Trust
- Stress reduction

# Teacher opinion

## **Formative intentionality**

*The use of a learning portfolio, that pretends to be reflexive, shows, in particular to the students, what we think and do in mathematics. It is considered that reflection, assessment, learning and the use of portfolio may be closely related in order to promote the continuous improvement of the student learning, the teacher practice, and a culture of self-assessment*

**Final classification:** using assessment criteria

## Study 3 – Individual work plan (Santos, 2016)

Context: Initiative of two mathematics teachers  
Collaborative work

Participants: Seven classes of 7<sup>th</sup> and 8<sup>th</sup> grades (students with 12 and 13 years old)

Focus: knowledge, problem solving

Strategy: 1<sup>st</sup> phase an individual written test in class; 2<sup>nd</sup> phase outside the classroom, solving a new set of questions with the opportunity of several feedbacks

Data collection: Students questionnaire, interviews to a group of students and teachers, documental analysis

## Results: Contributions for learning

- To develop reflection and self-regulation
  - To develop responsibility and persistency
  - To develop habits of work
- 
- Variable level of adhesion throughout the school year

## Contributions for a new culture of assessment

- Trust
- Stress reduction

# Teacher opinion

## Formative intentionality

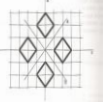
*Most of the students do not reveal great improvement in work habits and suitable methods to their difficulties (...) the analysis of the results show us the need to improve the strategy and to look to differentiate the supports, according to the needs showed by the students*

**Final classification:** similar to a test, analytical approach

# Which differences?

Test in  
two  
phases

3. Observa agora a figura seguinte, formada por quatro quadriláteros. A traço fino estão indicados os eixos de simetria da figura, a, b, c, e d. Os eixos de simetria e a quadrícula não fazem parte da figura.




a) Em que local poderás acrescentar um ponto A a esta figura de modo que a nova figura (formada pelos quatro quadrados e pelo ponto A) tenha os mesmos eixos de simetria?  
Marca o ponto na figura e explica o teu raciocínio.

b) Se para além do ponto A quiseres acrescentar um ou mais pontos, mas não sobre os quadrados, onde deves marcá-lo ou marcá-los para que os eixos de simetria se mantenham?  
Utiliza a mesma figura e explica a tua ideia.

Portfolio

A interseção é um ponto (ao lado fiz um esboço de um plano, com uma reta a interseção-lo, onde assinala o ponto de interseção). Portanto estamos à procura das coordenadas desse ponto.



A interseção é um ponto porque o vector director da recta não é perpendicular com o vector normal do plano.  
Portanto estamos à procura das coordenadas desse ponto.

Indiv.  
work  
plan

9.1)  $7^4 \times \left(-\frac{5}{2}\right)^4 \div \left(-\frac{5}{14}\right)^{-6}$

$$= 2^2 \times \left(-\frac{5}{2}\right)^4 \div \left(-\frac{5}{14}\right)^{-6}$$

$$= \left(\frac{1}{2}\right)^4 \times \left(-\frac{5}{2}\right)^4 \div \left(-\frac{5}{14}\right)^{-6}$$

$$= \left(\frac{-5}{14}\right)^4 \div \left(-\frac{5}{14}\right)^{-6}$$

$$\left(-\frac{5}{14}\right)^{10}$$

Levels of  
students'  
responsibility

Contexts

Teacher's role

Moments

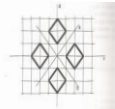
Proximity to  
usual  
practices



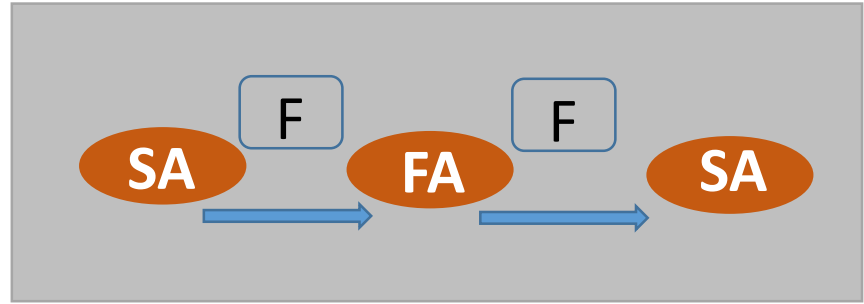
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Test in two phases

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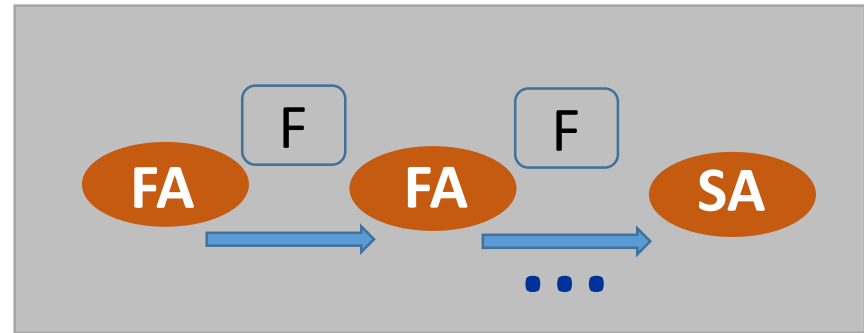


Portfolio

A interseção é um ponto (ao lado fiz um esboço de um plano, com uma reta a intersectá-lo, onde assinala o ponto de interseção). Portanto estamos à procura das coordenadas desse ponto.

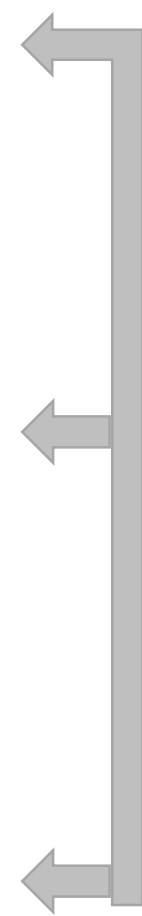
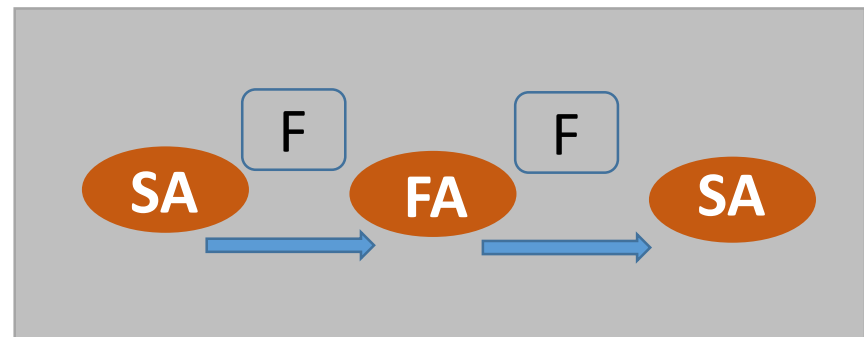


A interseção é um ponto porque o vector diretor da recta não é perpendicular com o vector normal do plano.  
 Portanto estamos à procura das coordenadas desse ponto.



Indiv. work plan

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 $\left(-\frac{5}{14}\right)^0$



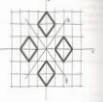
Moments



# Which similarities?

Test in  
two  
phases

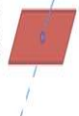
3. Observa agora a figura seguinte, formada por quatro quadriláteros. A traço fino estão indicados os eixos de simetria da figura, a, b, c, e d. Os eixos de simetria e a quadrícula não fazem parte da figura.



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 $= \left(\frac{1}{7}\right)^4 \times \left(-\frac{5}{2}\right)^4 \div \left(-\frac{5}{14}\right)^{-6}$   
 $= \left(\frac{5}{14}\right)^4 \div \left(-\frac{5}{14}\right)^{-6}$   
 $\left(-\frac{5}{14}\right)^0$

In different moments,  
different purposes

Promotes self  
assessment

Gives time to the  
students

Curriculum  
articulation

Permits  
differentiation

An intentional articulation between SA and FA

# Conclusions

# Principles to articulate summative and formative assessments

- To articulate means to establish relation and **not simultaneity** of processes
- To align **SA and FA** between each other, and with **teaching and curriculum**
- To consider the **culture of assessment** in the classroom and in other contexts where assessment occurs
- To reduce FA's formal strategies to articulate with SA do not eliminate **others strategies of FA** in other moments

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