

Using formative assessment in helping students understand geometrical concepts: The case of area

POLYXENI TSITSA , SECONDARY MATHEMATICS TEACHER



GROUP A , ATHENS – FORMAS IMPLEMENTATION 2019-2020
NATIONAL AND KAPODISTRIAN UNIVERSITY OF ATHENS -
DEPARTMENT OF MATHEMATICS

In the context of **FORMAS** implementation in Greece I participated in a professional development course for mathematics teachers working in lower secondary schools at the **National and Kapodistrian University of Athens** under the coordination of the Professors **Theodossios Zachariades** and **Giorgos Psycharis**.

The project presented here was designed during this course and was implemented at the **9th Junior High School of Acharnes** with students in the **8th grade**.

MATHEMATICS UNIT

✓ GEOMETRY - AREA OF 2D SHAPES



OBSERVATION NOTES

- ✓ Students have difficulty calculating areas, even when they know the formulas.



ACTIONS

- ✓ Design and use of **formative assessment activities** in order to diagnose students' needs and especially to clarify concepts and methodologies after teaching areas of 2D shapes (in particular: square, rectangle, parallelogram, rhombus, triangle, trapezium).
- ✓ Application of differentiated teaching to support students' learning.

THREE FORMATIVE ASSESSMENT ACTIVITIES

The following presentation concerns three formative assessment activities that were designed for a project of the course and were used in class.

CORRECTION OF A TEST

FORMATION OF ASSESSMENT CRITERIA



PEER-ASSESSMENT

WITH RANDOM SELECTION OF WORKSHEETS



SELF-ASSESSMENT



1st ACTIVITY OF FORMATIVE ASSESSMENT



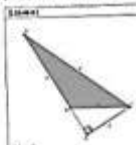


Students are asked to

- Correct a written test of an imaginary classmate.
- Create assessment criteria for that test.

ONCA: _____
 ESTADOS: _____

PERFORMANCE DAS EMBAIXAS EMBAIXADE - M3

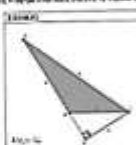
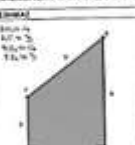

1. De acordo com o texto, qual a área da região hachurada? Para cada um dos casos, descreva o método utilizado para obter a resposta.

EMBAIXA	EMBAIXADA	EMBAIXADO
		
Área = $\frac{1}{2} \times 4 \times 3 = 6$ Área = $\frac{1}{2} \times 4 \times 3 = 6$ Área = $\frac{1}{2} \times 4 \times 3 = 6$	Área = $\frac{1}{2} \times (4+6) \times 3 = 15$ Área = $\frac{1}{2} \times (4+6) \times 3 = 15$ Área = $\frac{1}{2} \times (4+6) \times 3 = 15$	Área = $4 \times 3 = 12$ Área = $4 \times 3 = 12$ Área = $4 \times 3 = 12$
FORMULA: $A = \frac{1}{2} \times b \times h$	FORMULA: $A = \frac{1}{2} \times (b_1 + b_2) \times h$	FORMULA: $A = b \times h$
ERRORES: $A = \frac{1}{2} \times 4 \times 3 = 6$	ERRORES: $A = \frac{1}{2} \times (4+6) \times 3 = 15$	ERRORES: $A = 4 \times 3 = 12$
EMBAIXA: $A = \frac{1}{2} \times 4 \times 3 = 6$	EMBAIXA: $A = \frac{1}{2} \times (4+6) \times 3 = 15$	EMBAIXA: $A = 4 \times 3 = 12$

ONCA: _____
 ESTADOS: _____

PERFORMANCE DAS EMBAIXAS EMBAIXADE - M3

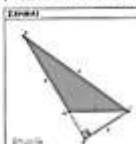
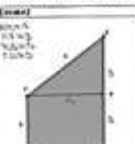

1. De acordo com o texto, qual a área da região hachurada? Para cada um dos casos, descreva o método utilizado para obter a resposta.

EMBAIXA	EMBAIXADA	EMBAIXADO
		
Área = $\frac{1}{2} \times 4 \times 3 = 6$ Área = $\frac{1}{2} \times 4 \times 3 = 6$ Área = $\frac{1}{2} \times 4 \times 3 = 6$	Área = $\frac{1}{2} \times (4+6) \times 3 = 15$ Área = $\frac{1}{2} \times (4+6) \times 3 = 15$ Área = $\frac{1}{2} \times (4+6) \times 3 = 15$	Área = $4 \times 3 = 12$ Área = $4 \times 3 = 12$ Área = $4 \times 3 = 12$
FORMULA: $A = \frac{1}{2} \times b \times h$	FORMULA: $A = \frac{1}{2} \times (b_1 + b_2) \times h$	FORMULA: $A = b \times h$
ERRORES: $A = \frac{1}{2} \times 4 \times 3 = 6$	ERRORES: $A = \frac{1}{2} \times (4+6) \times 3 = 15$	ERRORES: $A = 4 \times 3 = 12$
EMBAIXA: $A = \frac{1}{2} \times 4 \times 3 = 6$	EMBAIXA: $A = \frac{1}{2} \times (4+6) \times 3 = 15$	EMBAIXA: $A = 4 \times 3 = 12$

ONCA: _____
 ESTADOS: _____

PERFORMANCE DAS EMBAIXAS EMBAIXADE - M3

1. De acordo com o texto, qual a área da região hachurada? Para cada um dos casos, descreva o método utilizado para obter a resposta.

EMBAIXA	EMBAIXADA	EMBAIXADO
		
Área = $\frac{1}{2} \times 4 \times 3 = 6$ Área = $\frac{1}{2} \times 4 \times 3 = 6$ Área = $\frac{1}{2} \times 4 \times 3 = 6$	Área = $\frac{1}{2} \times (4+6) \times 3 = 15$ Área = $\frac{1}{2} \times (4+6) \times 3 = 15$ Área = $\frac{1}{2} \times (4+6) \times 3 = 15$	Área = $4 \times 3 = 12$ Área = $4 \times 3 = 12$ Área = $4 \times 3 = 12$
FORMULA: $A = \frac{1}{2} \times b \times h$	FORMULA: $A = \frac{1}{2} \times (b_1 + b_2) \times h$	FORMULA: $A = b \times h$
ERRORES: $A = \frac{1}{2} \times 4 \times 3 = 6$	ERRORES: $A = \frac{1}{2} \times (4+6) \times 3 = 15$	ERRORES: $A = 4 \times 3 = 12$
EMBAIXA: $A = \frac{1}{2} \times 4 \times 3 = 6$	EMBAIXA: $A = \frac{1}{2} \times (4+6) \times 3 = 15$	EMBAIXA: $A = 4 \times 3 = 12$

ACTIVITY

For the sake of the classroom dialogue and the simulation in real conditions, three different work samples with the most characteristic errors are given to the students.

The activity aims (through the change of roles)

- to arouse the interest of the students,
- to involve them in the process,
- to make them ask their own questions,
- to help them identify shapes and use the appropriate formula to calculate the area,
- to help them decide on the correctness of the solutions,
- to make them think about the necessity of having assessment criteria,
- to encourage them to collaborate.

For the teacher the activity is an opportunity to identify learning gaps, mis-conceptions and the students' needs.

ACTIVITY

Students try to

- identify errors,
- find the correct answers,
- justify their corrections,
- suggest assessment criteria,

They work in groups of two.



There is a differentiation in the individual assistance provided.

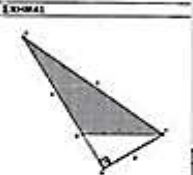
Guiding questions (e.g. What angle does the height form with the corresponding base?), suggestions (e.g. You can rotate the page and see the shape at a different angle.), worksheets with auxiliary material (shapes, area formulas).

HIGHLIGHTS OF THE ACTIVITY

- ✓ The way of working and the mistakes of the students are revealed to a great extent.

**MISTAKES - MISCONCEPTIONS
ΛΑΘΗ - ΠΑΡΕΞΗΓΗΣΕΙΣ**

ΕΡΩΜΑ1

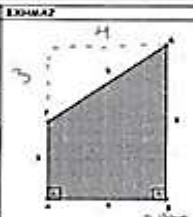


ΕΡΩΜΑ	πλάτος 4, ύψος 3 ✓
ΠΕΡΙΜΕΤΡΟΣ	$\pi = 4 + 5 + 3 = 12$ ✓
ΕΡΩΜΑ2	$S = \frac{4 \cdot 3}{2} = \frac{12}{2} = 6$

ΛΑΘΗ ΠΑΡΕΞΗΓΗΣΕΙΣ

Incorrect answer
The student did not identify correctly the height that correspond to the base/side 6

ΕΡΩΜΑ2

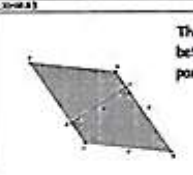


ΕΡΩΜΑ	πλάτος 4, ύψος 3 ✓
ΠΕΡΙΜΕΤΡΟΣ	$\pi = 4 + 5 + 3 = 12$ ✓
ΕΡΩΜΑ3	$S = \frac{(4+5) \cdot 3}{2} = \frac{27}{2} = 13.5$

ΛΑΘΗ ΠΑΡΕΞΗΓΗΣΕΙΣ

The student can not identify a trapezium, but he calculated correctly the area by using analysis/synthesis of known shapes.

ΕΡΩΜΑ3



The student can not distinguish between a rhombus on a plane parallelogram.

ΕΡΩΜΑ	πλάτος 5, ύψος 3 ✓
ΠΕΡΙΜΕΤΡΟΣ	$\pi = 5 + 6 + 3 + 5 = 19$
ΕΡΩΜΑ4	$S = 5 \cdot 6 = 30$

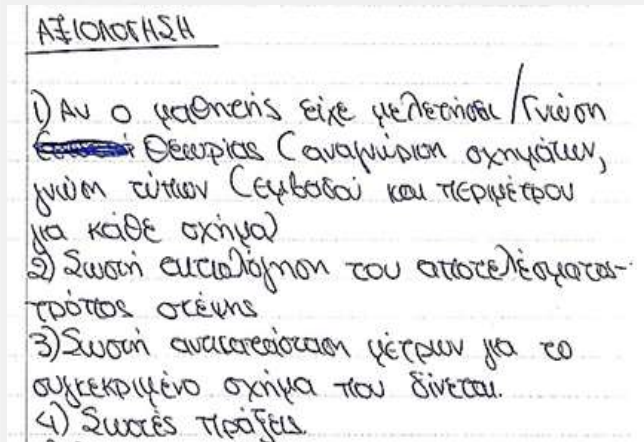
ΛΑΘΗ ΠΑΡΕΞΗΓΗΣΕΙΣ

The student makes incorrect application of the formula of the area by using two sides instead of a side and the corresponding height.

- The teacher records her observations of students' work during and after the end of the activity. That gives her a better picture of what the student know and can do.

HIGHLIGHTS OF THE ACTIVITY

- ✓ Creating assessment criteria helps the students in two ways:
 - To understand their mistakes concerning the calculation of the area.
 - To know what they have to do while solving the exercise.



The student

- Has studied.
- Can identify the shape.
- Uses the correct formula.
- Does not make mistakes when calculating.
- Identifies correctly the corresponding height.
- Explains and justifies correctly the results.

- Students gain a better understanding of the requirements expected and improve their knowledge on area of 2D-shapes.

2nd ACTIVITY OF FORMATIVE ASSESSMENT



Peer-assessment with random selection of worksheets.

ACTIVITY

Students are given an individual worksheet to find the areas of 2D shapes. It contains an exercise from the textbook with a complex shape.

All worksheets are put in a box (with the student's name hidden).

The correct solutions are presented by students.

Visual aid is provided by coloring the required shapes on the (interactive) whiteboard.

Each student chooses randomly a worksheet from the box.

The student corrects, assesses, grades the exercises and gives feedback.

ΣΧΗΜΑ	ΕΜΒΑΔΟΝ
✓ ABHO	$E = 3^2 = 3 \cdot 3 = 9 \text{ cm}^2$
✓ ΑΓΖΘ	$E = 6 \cdot 3 = 18 \text{ cm}^2$ <small>Κατά το δάπεδο όπως είναι το εμβαδόν.</small>
✓ ΑΓΕΗ	$E = 6 \cdot 3 = 18 \text{ cm}^2$
✗ ΑΒΕΘ	$E = \frac{(8+3) \cdot 3}{2} = 18 \text{ cm}^2$ <small>πρόσθεσε τις πλευρές στο ύψος.</small>
✓ ΑΔΖΗ	$E = \frac{(9+3) \cdot 3}{2} = 18 \text{ cm}^2$
✗ ΒΔΕ	$\frac{(8+8) \cdot 0}{2} + \frac{(3+6) \cdot 3}{2} = \frac{18 \cdot 3}{2} = \frac{54}{2} = 27$ $\frac{3 \cdot 6}{2} = 9 \text{ cm}^2$
✗ ΒΕΘ	$E = \frac{6 \cdot 0}{2} = \frac{6 \cdot 3}{2} = \frac{18}{2} = 9 \text{ cm}^2$ $\frac{3 \cdot 3}{2} = 4,5$
✗ ΑΒΕΗ	$E = 8 \cdot 0$ <small>$E = \frac{(6+6) \cdot 3}{2} = \frac{36}{2} = 18$</small> $6 \cdot 3 = 18 \text{ cm}^2$ <small>Κατά ένα άνω τμήμα</small>

HIGHLIGHTS OF THE ACTIVITY

- ✓ Some students find the requested area by adding or subtracting areas of basic 2D shapes (such as squares, rectangles or triangles) and not by using the area formula of one shape (in this case, a parallelogram or a trapezium). They develop strategies and make suggestions to fellow-students.

1. Να βρεις το εμβαδόν των παρακάτω σχημάτων:

ΣΧΗΜΑ	ΕΜΒΑΔΟΝ
ΑΒΘ	$3 \cdot 3 = 9$
ΑΓΖΘ	$6 \cdot 3 = 18$ 2 τετράγωνα
ΑΓΕΗ	$6 \cdot 3 = 18$ Ένα τετράγωνο 9 + 2 μικρά τετράγωνα 9 + 9 = 18
ΑΒΕΘ	$6 \cdot 3 = 18$ $\frac{12}{2} = 9$ $9 + 9 = 18$ 1 τετράγωνο με το κομμάτι του 9 και του 9 = 2 τετράγωνα
ΑΔΖΗ	2 τετράγωνα $9 + 9 = 18$ <i>etc</i>
ΒΔΕ	= με ένα τετράγωνο $3 \cdot 3 = 9$
ΒΕΘ	2,5 $\Theta Η \Theta$ μικρό τετράγωνο = 4,5 $9 + 4,5 = 13,5$ Ανα παρατήρησε το κομμάτι από 3 Ανα παρατήρησε το τετράγωνο από 2 και 9

Πρέπει να ανακαλύψουμε περισσότερο και να διαβιβάσουμε τους εινους και εμβαδους. Μπορεί να ειπεί και να πάρει και σχήματα τους σε βολήσια.

You must try more and study the formulas. You can cut and supplement the shapes. It might help you.

- Students become actively involved.

HIGHLIGHTS OF THE ACTIVITY

- ✓ The feedback comments given by the students indicate a reflection on their individual learning.

Διακρίνω πως γυρίζεις τους τύπους των εμβαδών, βέβαια, κάποιες φορές μπλοκάρω. Θα σου σύστηνα να ξαναδιαβάσεις τους τύπους των εμβαδών και να είσαι πιο προσεκτικός στις πράξεις. Τέλος, προσπάθησε να ρυθμίσεις τον χρόνο σου έτσι ώστε να σου είναι επαρκής!

I can see that you know the formulas of areas, but sometimes you get confused. I would recommend you to study again the formulas and to be more care ful when calculating. Finally, try to adjust your time so that it is sufficient for the exercises.

Ποτέ μην γράφεις εξήγηση όταν χρησιμοποιείς διαφορετική από τον κύριο μέθοδο για την επίλυση των προβλημάτων.

Explain what you do when you do not use the formula

ΠΡΟΣΠΑΡΕΙ ΝΑ ΠΡΟΒΕΞΕΙΣ ΕΛΑΧΙΣΤΑ ΤΟΥ ΠΟΛΥΠΛΗΘΥΝΣΜΟΥ

Try to avoid the multiplication errors.

2. Πιστεύεις ότι οι δράσεις αξιολόγησης γραπτών άλλων μαθητών βοήθησαν τη μάθησή σου;

Με βοήθησε πολύ για η διαδικασία της διόρθωσης ενός διαφορετικού ανθρώπου
κι βοήθησε και εγώ να καταλάβω πως να διορθώνω μικρά αλλά ευχάριστα λάθη

2. Do you think that the assessment of other students' worksheets helped your learning?

The process of correcting another person's worksheet helped me a lot to figure out how to correct small but important mistakes.

- Students like the assessment of other students' work, the fact that their mistakes are made by classmates too and the opportunity to offer feedback.

HIGHLIGHTS OF THE ACTIVITY

- ✓ The teacher's suggestion to color each shape (“following” the vertices) to see it clearly.

1. Ποια βοήθεια, από αυτές που σου δόθηκαν, σου ήταν πιο χρήσιμη:

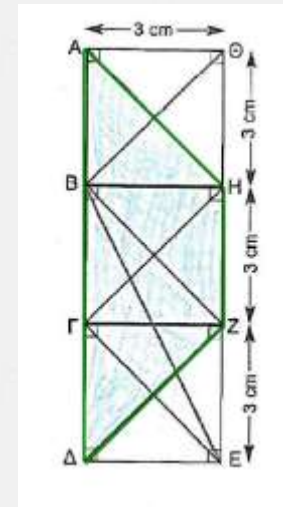
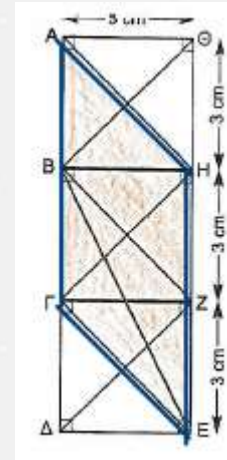
Βοήθεια με τα χρωματισμένα σχήματα, ώστε να ξεχωρίσω τα ~~σχήματα~~ να ~~βρω~~ βρω το σχήμα θα να υπολογίσω την βάση το ύψος.

1. Which help, from these given to you, was most useful to you?

The one with the colored shapes so that I can find in order to calculate the base, the height

1. Να βρεις το εμβαδόν των παρακάτω σχημάτων:

ΣΧΗΜΑ	ΕΜΒΑΔΟΝ
ΑΒΗΘ	$E = b \cdot u$ ($E = a^2$) $E = 3 \cdot 3 = 9 \text{ cm}^2$ ✓
ΑΓΖΘ	$E = b \cdot u$ $E = 6 \cdot 3$ ✓ $E = 18 \text{ cm}^2$
ΑΓΕΗ	$E = b \cdot u$ $E = 6 \cdot 3$ ✓ $E = 18 \text{ cm}^2$
ΑΒΕΘ	$E = \frac{(b_1 + b_2) \cdot u}{2}$ $E = \frac{(3 + 3) \cdot 3}{2}$ $E = \frac{12 \cdot 3}{2} = \frac{36}{2} = 18 \text{ cm}^2$
	$E = (P + R) \cdot u$



3rd ACTIVITY OF FORMATIVE ASSESSMENT



Self-assessment

ACTIVITY

Students are given an individual worksheet containing three exercises (designed by the teacher after taking in consideration the two previous activities).

- In each exercise students have to
- calculate the perimeter and the area of a 2D-shape and/or a (corresponding) height/base,
 - replace the correct elements in the formula (corresponding base/ height),
 - perform arithmetic operations and verify the results,
 - develop criteria to assess their work and decide on the next learning steps.

Students work individually at their own pace with minimum help.



ΟΝΟΜΑ: ΕΠΙΘΥΝΩ	ΤΜΗΜΑ ΗΜΕΡΟΜΗΝΙΑ		FIRST NAME LAST NAME	CLASS DATE
-------------------	---------------------	--	-------------------------	---------------

ΠΕΡΙΜΕΤΡΟΣ ΚΑΙ ΕΜΒΛΑΔΟΝ ΣΧΗΜΑΤΟΣ - ΘΕ 3

ΑΣΚΗΣΗ 1

(α) Το σχήμα ΑΒΓΔ είναι:
 (β) Η περιφέρεια του ΑΒΓΔ είναι:
 (γ) Το εμβαδόν του ΑΒΓΔ είναι:
 (δ) Το ύψος x που αντιστοιχεί στην πλευρά ΑΔ είναι:

ΑΣΚΗΣΗ 2

(α) Το σχήμα ΕΖΗΘ είναι:
 (β) Το εμβαδόν του ΕΖΗΘ είναι:
 (γ) Η πλευρά ΕΖ είναι:
 (δ) Η περιφέρεια του ΕΖΗΘ είναι:

ΑΣΚΗΣΗ 3

(α) Το σχήμα ΑΒΓ είναι:
 (β) Η περιφέρεια του ΑΒΓ είναι:
 (γ) Το εμβαδόν του ΑΒΓ είναι:
 (δ) Το ύψος ΑΔ που αντιστοιχεί στην πλευρά ΒΓ είναι:

EXERCISE 1

(a) The ΑΒΓΔ shape is:
 (b) The perimeter of the ΑΒΓΔ is:
 (c) The area of ΑΒΓΔ is:
 (d) The height x corresponding to the ΑΔ side is:

EXERCISE 2

(a) The ΕΖΗΘ shape is:
 (b) The area of the ΕΖΗΘ is:
 (c) The side ΕΖ is x:
 (d) The perimeter of the ΕΖΗΘ is:

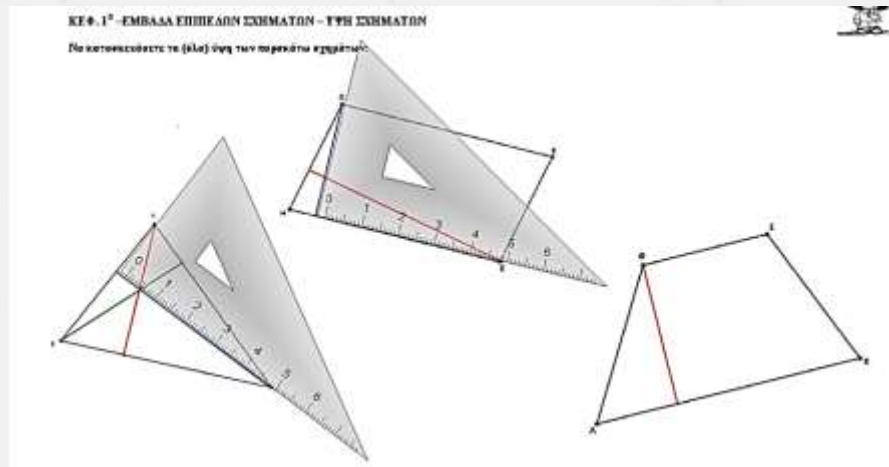
EXERCISE 3

(a) The ΑΒΓ shape is:
 (b) The perimeter of the ΑΒΓ is:
 (c) The area of the ΑΒΓ is:
 (d) The height ΑΔ = x corresponding to the side ΒΓ is:

- The teacher adjusts the time, the instructions and the activities accordingly.
- It takes more time to work in that way.

HIGHLIGHTS OF THE ACTIVITY

- ✓ Some students continue to confuse the shorter side of a parallelogram with the distance between its parallel sides (height).



The teacher adjusts the activities so that those students practice by constructing altitudes.

- For further implementation:
An interactive application is needed for the students to test if a height correspond to a particular side, in order to achieve understanding.

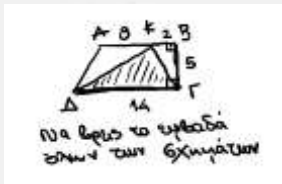
HIGHLIGHTS OF THE ACTIVITY

Students

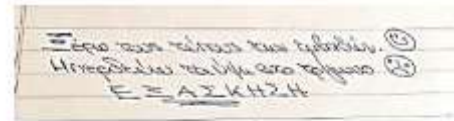
- ✓ estimate how well they did,
- ✓ assess their own work and create assessment criteria,
- ✓ reflect on the work's strong and weak points, write down the difficulties they encountered,
- ✓ make their learning plan,
- ✓ gain confidence.

ADJUSTING ACTIVITY

The teacher asks the students to create their own problem involving shape area (with at least two different shapes).



- Students deepen their understanding of the subject and tighten their learning.



Απέχει να κερδίσει ότι σε ένα ορθό τρίγωνο, το ύψος είναι 2 βάσεις.

I have to understand that a shape has at least two heights and two bases.

Επίσης, όταν κάποιος κάνει λάθος, (smiley face)
 Η προσπάθεια να λύσει ένα πρόβλημα, (sad face)
 E = A + K + H + H
 Απέχει να κερδίσει ότι σε ένα ορθό τρίγωνο, το ύψος είναι 2 βάσεις.
 Επίσης, όταν κάποιος κάνει λάθος, (smiley face)
 Η προσπάθεια να λύσει ένα πρόβλημα, (sad face)

Yes I can assess the work I have done.
 At first I check the formula (if it is the right one)
 Then the operations
 The way I have written (presented) them.
 I try the same exercise again, until I figure it out and solve it correctly

Επίσης, όταν κάποιος κάνει λάθος, (smiley face)
 Η προσπάθεια να λύσει ένα πρόβλημα, (sad face)
 E = A + K + H + H
 Απέχει να κερδίσει ότι σε ένα ορθό τρίγωνο, το ύψος είναι 2 βάσεις.
 Επίσης, όταν κάποιος κάνει λάθος, (smiley face)
 Η προσπάθεια να λύσει ένα πρόβλημα, (sad face)

I think I can solve an exercise about (calculating) areas because I know the formulas well.
 I believe I am in a certain point where I can calculate the area of a shape, because I have managed to calculate areas of complex shapes and I know the way (how) to do it.

THE END



THANK YOU!

ΕΥΧΑΡΙΣΤΩ!

