The influence of cognitive style on children's ability to solve reflection and rotation tasks

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Abstract

This study aimed to explore nine to fourteen year old students' cognitive style and its relation to the ability of solving transformation geometry tasks of reflection and rotation. Cognitive style was measured based on the object-spatial-verbal model proposed by Blazhenkova and Kozhevnikov (2009). Our objective was to investigate whether students' cognitive style is related to their abilities in reflection and rotation tasks. 501 primary and secondary school students were given the following instruments: a) a Greek translation of the children's version of the self-report Object-Spatial Imagery and Verbal Questionnaire (c-OSIVQ) (Blazhenkova, Becker, & Kozhevnikov, 2011), to assess cognitive style. The questionnaire included three scales assessing the object, spatial and verbal cognitive styles. Each scale consisted of 15 self-rating statements. The object, spatial and verbal ratings were averaged to create corresponding scores, and b) a transformation geometry test, to assess ability in tasks of reflection and rotation. Four types of tasks were included for each type of transformation: 1) recognition of the image of a reflection/rotation, 2) recognition of a reflection/rotation, 3) construction of the image for a given reflection/rotation, and 4) identification of the parameters (relation between image and pre-image) of a given reflection/rotation. The scores were calculated for each type of transformation as well as students' overall ability. The findings confirmed the reliability of the questionnaire (α=0.90). The results suggest that children's mean scores in the three scales decrease with age, while their ability in the transformation geometry tasks increases. Verbal score was negatively correlated to ability in reflection and rotation tasks, and to overall ability ($p<0.05$). Object score was negatively related only to overall ability ($p<0.05$). Stepwise regression analyses with spatial, object and verbal scores as depended variables, confirmed that spatial and verbal scores are predictors for the overall ability in the transformational geometry test ($p<0.05$). Studies in mathematics education call for a clarification between the relationship of learners' preference for visual information processing and their abilities in mathematics (Pitta-Pantazi & Christou, 2009; Presmeg, 2006) and recent studies in the field tend to rely on the distinction between two types of visualizers – spatial and object to examine individual differences in various mathematical concepts (Xistouri & Pitta-Pantazi, 2011). This study contributes to the field by examining the impact of elementary and secondary students' cognitive style in transformation geometry tasks ability and suggests that verbal and object imagery abilities are negatively related to transformation geometry ability. In addition, a growing body of research is currently advocating the potential of styles to impact on performance in education (Evans & Cools, 2011). Our results about the relationship between the object, spatial and verbal cognitive styles and transformation geometry performance provide more evidence for this perspective. Our findings also raise the issue that a number of learners may face difficulties in transformational geometry due to their cognitive styles.

Keywords: cognitive style, object, spatial, transformation geometry

1. Introduction

Research in the area of mathematics education, during the last few years, shows an increasing emphasis on students' understanding of transformation geometry (Hollebrands, 2003; Naidoo, 2010; Portnoy, Grundmeier, & Graham, 2006; Poswolsky, 2006; Yanik &

Flores, 2009). The main reasons for this emphasis are, the close connection between transformation geometry and children's development of geometric and spatial thinking (Hollebrands, 2003) and also, its relation to a range of activities in academic and every-day life, such as geometrical constructions, art, architecture, carpentry, electronics, mechanics, clothing design, geography and navigation (Boulter & Kirby, 1994). Transformation geometry is the kind of geometry that refers to the mental or physical transformation of shapes. The most common types of geometric transformations in literature and in primary school textbooks are translation, axial reflection and point rotation. This paper focuses on the latter types, namely axial reflection and point rotation, which in this paper are referred to as reflection and rotation.

Performance in geometric transformations has been previously connected to the holistic-analytic types of processing (Boulter & Kirby, 1994) and visual-analytic strategies (Naidoo, 2010). It has also been connected to spatial ability (Dixon, 1995; Kirby & Boulter, 1999). However, despite the rather obvious relation between transformation geometry and visual imagery, there appear to be no studies relating it to the visual-verbal cognitive style proposed by Paivio (1971). A recent attempt seems to provide evidence for a relation between prospective teachers' performance in the geometric transformation of axial reflection and the three-dimensional cognitive style model proposed by Blazhenkova and Kozhevnikov (2009) that distinguishes between Object imagery, Spatial Imagery and Verbal dimensions (Xistouri & Pitta-Pantazi, 2011). The present study aspires to examine the relation between transformation geometry ability and the object-spatial-verbal cognitive style model in children. Specifically, the aim of the paper is to investigate the relationship between primary and secondary school students' cognitive style and their ability in transformation geometry tasks of reflection and rotation.

2. Literature review

2.1. Mathematics education research and cognitive styles

The notion of cognitive style has received wide attention in psychology research (for a review, see Rayner & Riding, 1997). It can be defined as "an individual's characteristic and consistent approach to organising and processing information" (Tennant, cited in Riding, 1997). Although there appear to be various conceptualisations of cognitive styles (for a classification, see Sternberg and Grigorenko, 1997), most of the researchers agree that cognitive style is a construct which is relatively stable over domain and time.

In mathematics education research, the majority of studies on individual differences seem to adopt the bipolar discrimination of verbal and visual cognitive style. However, it needs to be noted that this distinction was not referred to as "cognitive style" but as preferred type/mode of thinking, or type of students (Kruteskii, 1976; Lean & Clements, 1981;

Presmeg, 1986a, 1986b, 2006). The main idea emerging from these studies is that visual-spatial processes are distinct from verbal processes and that mathematics involves not only verbal processes but also visual reasoning (Presmeg, 1986a; Sfard, 1991).

Nevertheless, the results of the relationship between visualisation and mathematical performance are unclear (Pitta-Pantazi & Christou, 2009; Presmeg, 2006). Some studies found that visual–spatial memory is an important factor which explains the mathematical performance of students (Battista & Clements, 1998), while other studies showed that students classified as visualizers do not tend to be among the most successful performers in mathematics (Presmeg, 1986a). In the case of performance in transformation geometry, although it has not yet been linked to the verbalizers/visualizers distinction, it has often been connected to high spatial abilities (Dixon, 1995; Kirby & Boulter, 1999).

In their studies with adults, Blazhenkova and Kozhevnikov (2009) suggested that there exist two distinct imagery subsystems that help individuals process visual information in different ways. Specifically, they suggest that there is an object imagery system and a spatial imagery system. The object imagery system processes the "visual appearance of objects and scenes in terms of their shape, colour information and texture", while the spatial imagery system processes "object location, movement, spatial relationships and transformations and other spatial attributes of processing" (p. 1475). Thus, recent research identified two distinct types of visualizers. Object visualizers who use imagery to construct images of objects and process visual information globally and holistically as whole perceptual objects and spatial visualizers who use imagery to represent spatial relations, make complex spatial transformations and process visual images analytically and sequentially, part-by-part (Kozhevnikov, Kosslyn, & Shephard, 2005). Recently, Blazhenkova, Becker & Kozhevnikov (2011) have investigated and confirmed the existence of these three dimensions of cognitive style also in children between 8 and 17 years old, and related them to learning preference for various academic subjects. Learning preference for geometry was significantly correlated to the spatial scale.

2.2. Students' understanding of transformation geometry

Since the inclusion of transformation geometry in mathematics curricula in the early 70's, a growing emphasis has been raised around the importance of teaching and understanding geometric transformations (Yanik & Flores, 2009). Studies on transformation geometry focused mainly on the understanding of the concepts of transformations including translations, reflections, rotations and compositions of transformations (Boulter & Kirby, 1994; Edwards & Zazkis, 1993; Harper, 2002; Hollebrands, 2003; Law, 1991; Thaqi, Gimenez, & Rosich, 2011; Xistouri & Pitta-Pantazi, 2011). Although these studies focused on describing difficulties in the understanding of geometric transformations, only few of them examined children’s difficulties and even fewer have attempted to examine these difficulties from the scope of individuals’ different types of information processing.

Such studies viewed these differences in the sense of strategies applied when solving a task. Moreover, they related the different types of strategies to performance in geometric transformations tasks. For instance, Boulter and Kirby (1994) analyzed elementary school students’ strategies based on the holistic/analytic distinction, when solving geometric transformation tasks of reflections and rotations. Students’ strategies were classified as holistic when the task was solved with the visualization of the whole shape as an entity and as analytic when the shape was visually fragmented and transformed piece by piece. The results of this study indicated that some students showed preference for either the holistic or analytic processing, and that use of analytic strategies was associated with success. On the contrary, a recent study by Naidoo (2010) suggests that learners who have a visual (versus analytic) understanding could be better in understanding the effects of transformations on whole figures, rather than focusing on isolated points.

Nevertheless, it seems that it is not clear in the literature which type of processing, visual or analytic, can lead to better understanding and performance in geometric transformations, such as reflection and rotation. Since there are indications that spatial ability is positively related to performance in geometric transformations generally (Dixon, 1995; Kirby & Boulter, 1999) and that the spatial cognitive style is related to performance in the geometric transformation of axial reflection specifically (Xistouri & Pitta, 2011), we believe that the object-spatial-verbal cognitive style model proposed by Blazhenkova and Kozhevnikov (2009) can lead to a better understanding about the relation between different types of processing information and ability in transformation geometry, and specifically to the geometric transformations of reflection and rotation. Our hypothesis is that children’s performance in reflection and rotation will relate positively to the spatial imagery style.

3. Methodology

3.1. Participants

The participants were 501 (262 boys and 239 girls) primary and secondary school students. Specifically, 299 students came from primary school (91 fourth-graders, 115 fifth-graders, and 93 sixth-graders) and 202 came from secondary school (106 first-graders and 96 second-graders).

3.2. Procedure and materials

All the participants were administered a self-report questionnaire to assess their cognitive style and a geometry test to measure their ability to solve transformation geometry tasks of reflection and rotation.

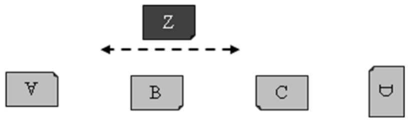
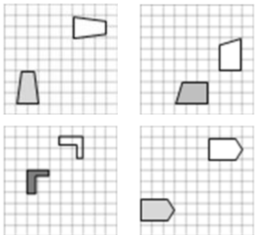
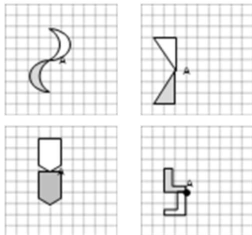
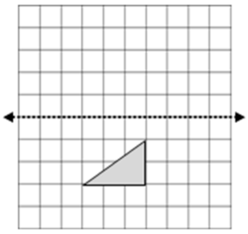
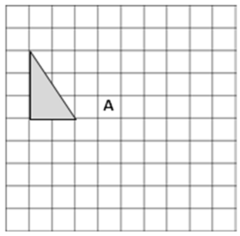
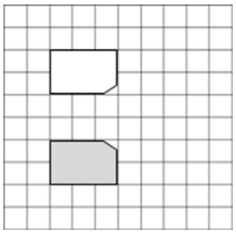
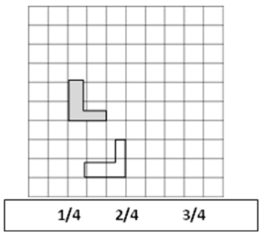
The self-report questionnaire was the children's Object-Spatial Imagery and Verbal Questionnaire (c-OSIVQ), developed by Blazhenkova, Becker, & Kozhevnikov (2011). It was translated in Greek for the needs of this study. The c-OSIVQ is used to assess individual differences in spatial imagery, object imagery and verbal cognitive style in children. The children were asked to read 45 statements and rate each item on a 5-point Likert scale with 1 indicating total disagreement and 5 total agreement. Ratings 2 to 4 indicated intermediate degrees of agreement/disagreement. Fifteen of the items measured object imagery preference and experiences, fifteen items measured spatial imagery preference and experiences and fifteen items measured verbal preference and experiences. The items assessing object-imagery cognitive style included statements about vividness of mental imagery, photographic memory, preferences for painting with colours, ease of image maintenance, and elicited imagery (for example "*When reading a book, I can usually imagine clear, colourful pictures of the people and places*". The items assessing spatial-imagery cognitive style included statements about 3D geometry, schematic mental imagery, mechanical inclination, and spatially intensive games (for example "*I am good at solving geometry problems with 3D figures*". Verbal cognitive style items were statements referring to the speed of reading, ease of writing, fluency in expressing thoughts and ideas verbally, and storytelling (for example "*I am good at expressing myself in writing*" (Blazhenkova, Becker, & Kozhevnikov, 2011, p.282).

The children were given this questionnaire during school-time. They were given approximately thirty minutes to complete the questionnaire. For each participant, the fifteen item ratings for each factor were averaged to create object imagery, spatial imagery and verbal scale scores.

The geometry test, which was developed for the needs of this study, aimed to assess the students' ability in transformation geometry tasks of reflection and rotation. It included the following four types of tasks for each type of transformation: 1) recognition of the image of a reflection/rotation (3 tasks for reflection and 3 tasks for rotation), 2) recognition of a reflection/rotation (4 tasks for reflection and 4 tasks for rotation), 3) construction of the image for a given reflection/rotation (5 tasks for reflection and 5 tasks for rotation), and 4) identification of the parameters (relation between image and pre-image) of a given reflection/rotation (4 tasks for reflection and 4 tasks for rotation). Examples of the tasks are given in Table 1.

The children were administered this test during school time. They were given approximately two thirty-minute sessions to complete the tasks. The coding for the geometry test was 0 for each incorrect response and 1 for each correct response in the case of type 1 and type 2 tasks, which were multiple choice tasks. Partial credit between 0 and 1 was given to type 3 and type 4 tasks. The scores were calculated for each type of geometric transformation and for overall ability in the geometry test.

Table 1 – Types of tasks and examples for reflection and rotation

Type of task	*Example tasks for reflection*	*Example tasks for rotation*
1. Recognition of the image of a reflection/rotation	Which of the following shapes is the reflection of shape Z over a horizontal line of symmetry?  	Which of the following shapes is the rotation of shape Z at ¼ of a turn? 
2. Recognition of a reflection/rotation	Which of the following images presents the reflection of the shaded image? 	Which of the following images presents the translation of the shaded image? 
3. Construction of the image for a given reflection/rotation	Draw the reflection of each shape over the given line of symmetry. 	Rotate the shape ¾ of a turn to the right. 
4. Identification of the parameters (relation between image and pre-image) of a given reflection/rotation	Draw the line of symmetry for every case. 	Find the point of rotation and circle the fraction that shows how much the shape turned to the right. 

4. Results

The aim of this study was to explore primary and secondary school students' cognitive style and its relation to the ability in solving transformation geometry tasks of reflection and rotation. Cognitive style was measured based on the object-spatial-verbal model proposed by Blazhenkova and Kozhevnikov (2009). The c-OSIVQ questionnaire developed by Blazhenkova, Becker and Kozhevnikov (2011) was translated in Greek and was administered to 501 nine to fourteen year old students.

Since this study used a translation of the original version of the c-OSIVQ, we considered it important at first to confirm the internal reliability of the Greek translated version of the questionnaire. For this purpose, the Cronbach's α coefficient of each scale and of the overall questionnaire were calculated. For the object scale Cronbach's α was 0.82, for the spatial scale Cronbach's α was 0.83 and for the verbal scale Cronbach's α was 0.84. The overall reliability of the questionnaire was α=0.90. According to Nunnaly (1978), values of α above 0.7 are acceptable. Hence, these results confirm the reliability of the questionnaire.

Table 2 –Means and standard deviations for each grade in the spatial, object and verbal scale and in the transformation geometry test

Cognitive style scale/ Geometry ability	*Primary Grade 4*		*Primary Grade 5*		*Primary Grade 6*		*Secondary Grade 1*		*Secondary Grade 2*	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Object scale score	4.17	0.63	3.81	0.64	3.69	0.62	3.77	0.62	3.45	0.67
Spatial scale score	3.49	0.75	3.36	0.80	3.29	0.81	3.39	0.70	3.10	0.70
Verbal scale score	3.99	0.64	3.57	0.73	3.39	0.65	3.55	0.64	3.01	0.63
Ability in geometry	8.00	4.07	11.45	5.90	13.93	6.25	16.22	5.96	17.36	6.49

Table 2 presents the students' means and standard deviations in the three scales of the questionnaire and for the geometry test by grade. The values suggest that in all grades the students scored higher in the object scale and lower in the spatial scale, with the exception of Secondary Grade 2. According to Blazhenkova and Kozhevnikov (2009) however, it is common for people to rate theirselves higher on the object scale. Moreover, Table 2 also shows how students' mean ratings in the three scales tend to decrease through grade level, with some peaks in all cases in Grade 1 of Secondary School (around the age of 13). Our results are consistent with those of Blazhenkova, Becker and Kozhevnikov (2011) which suggest that cognitive style does not significantly change with age, however all scales appear to have a decrease between the age of 9 to 12 years old and then appear to peak around the ages of 13-14. Moreover, Table 2 shows that ability in transformation geometry increased significantly with age ($F(4,496)=39.21$, $p<0.01$), with post-hoc analyses showing that all differences are significant except the difference between Secondary Grade 1 and Secondary Grade 2. This finding is also consistent with Blazhenkova, Becker and Kozhevnikov (2011), who observed that cognitive abilities tend to undergo significant age-related changes, while cognitive styles do not change significantly with age.

Our first objective was to investigate whether primary and secondary school students' cognitive style is related to ability in reflection and rotation tasks. Bivariate correlation analyses were performed between spatial, object and verbal scores and the students' scores in the reflection and the rotation tasks separately, as well as their overall ability score in the test. Table 3 presents the Pearson's correlation coefficients between these variables.

Table 3 – Pearson's correlation coefficients between variables

	Reflection tasks score	*Rotation tasks score*	*Overall ability*
Object scale score	-0.089	-0.082	-0.092*
Spatial scale score	0.035	0.039	0.040
Verbal scale score	-0.147**	-0.106*	-0.136**

(*) p< 0.05, (**) p<0.01

As can be seen in Table 3, the verbal scale score was the only cognitive style scale score that was negatively related to all ability scores at level of significance p=0.01. Specifically, the verbal score was negatively correlated to ability in the tasks of reflection, ability in the tasks of rotation (r=-0.106, p<0.05) and to overall ability (r=-0.136, p<0.01). This means that the higher the verbal cognitive style one reports to have, the lower their performance will be in reflection and rotation tasks. Table 3 also shows that object score was negatively related at a significant level only to overall performance (r=-0.092, p<0.05). It is important to note that while not any ability's correlation with the spatial scale score is statistically significant, they are all positive. This indicates that the higher the spatial cognitive style dimension one reports to have, the higher their performance in reflection and rotation tasks is likely to be. However, further research is required to confirm this conjecture.

The second objective of this study was to investigate whether and which of the three cognitive style dimensions – spatial, object and verbal – can predict ability in reflection and rotation tasks. Based on the connection found in literature between spatial ability and transformation geometry ability in children and the relation between prospective teachers' spatial cognitive style and ability in axial reflection, our hypothesis was that performance in reflection and rotation tasks will be predicted by the students' spatial scale score. Stepwise multiple regression analysis with spatial, object and verbal scores as predictive variables were performed to confirm our hypothesis. Table 4 presents the results of the multiple regression.

Table 4 – Multiple regression analysis coefficients for overall ability in reflection and rotation tasks

	b	*p*	*T*
Spatial scale score	.10*	.04	2.02
Object scale score	-.68	.50	-.68
Verbal scale score	-.17**	.00	-3.52

(*) p<0.05, (**) p<0.01

The results presented in Table 4 confirm our hypothesis that overall ability in the reflection and rotation tasks can be significantly predicted by the students' spatial scale score. Specifically, spatial scale score can positively predict students' overall ability in transformation geometry tasks of reflection and rotation ($b= 0.10$, $p<0.05$). This finding is in line with the results of Xistouri & Pitta-Pantazi (2011). However, verbal scale score also appears to be a significant predictor for overall ability, but it is a negative predictor. Verbal scale score can negatively predict students' overall ability in transformation geometry tasks of reflection and rotation ($b= -0.17$, $p<0.01$). Object scale score was not found to be a statistically significant predictor.

5. Discussion

The aim of this paper was to explore primary and secondary students' cognitive style and its relation to the ability in solving transformation geometry tasks of reflection and rotation. It was based on the object-spatial-verbal model proposed by Blazhenkova and Kozhevnikov (2009). Our objectives were: 1) to investigate whether primary and secondary students' cognitive style is related to ability in reflection and rotation tasks, and 2) to explore which of the three dimensions predict this ability. In this section, the results of this study are discussed from an educational aspect. Additionally, some possible teaching implications and further research questions are addressed.

The results of this study confirm the reliability of the Greek translated version of the questionnaire. Moreover, our study suggests that even though the development of cognitive style and the cognitive ability of transformation geometry are closely related, their developmental trajectories seem to be different. According to Blazhenkova, Becker, and Kozhevnikov (2011, p.287), "cognitive style's development is separate from cognitive development, although they are highly interlinked". However, a limitation of the current study is the cross-sectional nature of our sample, and future longitudinal studies on the development of cognitive style and transformation geometry ability are required to fully understand this relationship, and the differences in the developmental courses that these constructs follow.

The object imagery style dimension seems to be negatively related to overall ability in reflection and rotation. This finding contributes to clarifying the relationship between mathematics ability and visual strength, by providing evidence that only object imagery abilities are negatively related to transformational geometry performance. Moreover, the verbal scale score was negatively related and has proved to be a significant predictor for overall ability in reflection and rotation tasks. This is an important issue, since it means that people who rate themselves higher in the verbal scale are less likely to succeed in solving reflection and rotation tasks. This raises some serious questions regarding students' understanding of geometric transformations in primary and secondary education. Our findings about the relationship between the object and verbal cognitive styles and transformation geometry performance provide more evidence and further support the perspective of current research which is advocating the potential of styles to impact on performance in education (Evans & Cools, 2011). It appears that a number of learners may face difficulties in transformation geometry due to their cognitive styles. Future studies may address this issue.

Spatial imagery scale score was found to be a significant positive predictor for overall ability. In a similar study with prospective teachers, spatial imagery was also found to be a significant positive predictor for the geometric transformation of reflection (Xistouri & Pitta-Pantazi, 2011). Transformation geometry tasks generally involve the mental or physical manipulation of shapes to new positions or orientations in space (Boulter & Kirby, 1994). It is possible that spatial visualizers have the flexibility to shift their attention in decomposing the shape into smaller parts and analytically locate the positions of the image in space for each part of the shape when transformed, without losing track. Further studies can qualitatively look into the different strategies used by spatial visualizers, object visualizers and verbalizers, in order to understand what kind of strategies they apply and what difficulties may be raised in their understanding due to their preferred mode of processing.

In summary, as we pointed out, there is a relation between students' cognitive style and their ability in transformation geometry tasks of reflection and rotation. Future studies can qualitatively examine the different strategies applied in solving axial reflection tasks by subjects with different cognitive styles. Such research could provide insight to mathematics educators on effective ways to promote understanding of transformation geometry concepts in primary and secondary school. Additionally, the current study supports the reliability of the children's Object-Spatial Imagery Verbal Questionnaire, and also the predictive nature of the spatial-object-verbal cognitive style model, by confirming that performance in a scientific matter of knowledge such as transformation geometry is related to a distinct kind of imagery, the spatial imagery cognitive style.

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6. References

Battista, M.T. & Clements, D.H. (1998). Finding the number of cubes in rectangular cube buildings. *Teaching Children Mathematics,* 4(5), 258-264.

Blazhenkova, O., Becker, M., & Kozhevnikov, M. (2011). Object–spatial imagery and verbal cognitive styles in children and adolescents: Developmental trajectories in relation to ability. *Learning and Individual Differences*, *21*(3), 281-287.

Blazhenkova, O. & Kozhevnikov, M. (2010). Visual-object ability: A new dimension of non-verbal intelligence. *Cognition*, 117 (3), 276-301.

Blazhenkova, O. & Kozhevnikov, M. (2009). The new object-spatial-verbal cognitive style model: Theory and measurement. *Applied Cognitive Psychology*, 23(5), 638-663.

Blazhenkova, O., Kozhevnikov, M. & Motes, M. (2006). Object-Spatial Imagery: A New Self-Report Imagery Questionnaire. *Applied Cognitive Psychology*, 20, 239–263.

Boulter, D. R. & Kirby, J. R. (1994). Identification of strategies used in solving transformational geometry problems. *Journal of Educational Research*, 87 (5), 298-303.

Chabris, C. F., Jerde, T. E., Woolley, A. W., Gerbasi, M. E., Schuldt, J. P., Bennett, S. L., et al.

(2006). Spatial and object visualization cognitive styles: Validation studies in 3800 individuals. Technical Report #2.

Dixon, J. K. (1995). *English language proficiency and spatial visualization in middle school students' construction of the concepts of reflection and rotation using the "The Geometer's Sketchpad.* Unpublished Doctoral dissertation, University of Florida.

Edwards, L. & Zazkis, R. (1993). Transformation geometry: Naive ideas and formal embodiments. *Journal of Computers in Mathematics and Science Teaching*, 12(2), 121-145.

Evans, C. & Cools, E. (2011). Applying styles research to educational practice. *Learning and Individual* Differences, 21, 249-254.

Harper, S.R. (2002), *Enhancing elementary pre-service teachers' knowledge of geometric transformations*. Unpublished Doctoral Dissertation, University of Virginia.

Hollebrands, K. (2003). High school students' understandings of geometric transformations in the context of a technological environment. *Journal of Mathematical Behavior*, 22 (1), 55-72.

Kirby, J.R. & Boulter, D.R. (1999). Spatial ability and transformational geometry. *European Journal of Psychology of Education*, 14 (2), 283-294.

Kozhevnikov, M., Blazhenkova, O. & Becker, M. (2010). Trade-off in Object versus Spatial Visualization Abilities: Restriction in the Development of Visual Processing Resources. *Psychonomic Bulletin & Review*, 17, 29-35.

Kozhevnikov, M., Hegarty, M. & Mayer, R.E. (2002). Revising the visualizer/verbalizer dimension: evidence for two types of visualizers. *Cognition & Instruction*, 20, 47-77.

Kozhevnikov, M., Kosslyn, S. & Shephard, J. (2005). Spatial versus object visualizers: A new characterization of visual cognitive style. *Memory and Cognition*, 33, 710-726.

Kruteskii, V.A. (1976). *The psychology of mathematical abilities in schoolchildren*. Chicago: University of Chicago Press.

Law, C.K. (1991). *A genetic decomposition of geometric transformation*, Unpublished Doctoral Dissertation: Purdue University.

Lean, G. & Clements, M.A., (1981). Spatial ability, visual imagery, and mathematical performance. *Educational Studies in Mathematics*, 12, 267-299.

Naidoo, J. (2010). Strategies used by Grade 12 mathematics learners in transformation geometry. In Mudaly V. (Ed.) *Proceedings of the Eighteenth Annual Meeting of the Southern African Association for Research in Mathematics, Science and Technology Education (Vol.2)*. Natal: University of Kwazulu, (pp. 182-190).

Nunnaly, J. (1978). *Psychometric theory*. New York: McGraw-Hill.

Paivio, A. (1971). *Imagery and verbal processes*. New York: Holt, Rinehart & Winston.

Parsons, R.R. (1993). *Teacher beliefs and content knowledge: influences on lesson crafting of pre-service teachers during geometry instruction*. Unpublished Doctoral Dissertation. Washington State University.

Pitta-Pantazi, D. & Christou, C. (2009). Cognitive styles, dynamic geometry and measurement performance. *Educational Studies in Mathematics*, 70, 5-26.

Poswolsky, K. (2006). *Transformation Geometry*. Unpublished Doctoral Dissertation. New York: Hofstra University.

Portnoy, N., Grundmeier, T. & Graham, K. (2006). Students' understanding of mathematical objects in the context of transformational geometry: Implications for constructing and understanding proofs. *Journal of Mathematical Behavior*, 25, 196-207.

Presmeg, N.C. (1986a), Visualization and mathematical giftedness. *Educational Studies in Mathematics*, 17, 297-311.

Presmeg, N.C. (1986b). Visualization in high school mathematics. *For the Learning of Mathematics*, 6 (3), 42-46.

Presmeg, N. (2006). Research on visualization in learning and teaching mathematics. In A. Gutiérrez & P. Boero (Eds.), *Handbook of Research on the Psychology of Mathematics Education: Past, Present and Future*. Rotterdam: Sense.

Rayner, S.G. & Riding, R.J. (1997). Towards a categorization of cognitive styles and learning styles. *Educational Psychology*, 17, 5-28.

Riding, R.J. (1997). On the nature of cognitive style. *Educational Psychology*, 17, 29-50.

Sfard, A. (1991). On the dual nature of mathematical conceptions: reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, 22, 1-36.

Sternberg, R.J. & Grigorenko, E.L. (1997). Are cognitive styles still in style? *American Psychologist*, 52 (7), 700–712.

Thaqi, X., Gimenez, J. & Rosich, N. (2011). Geometrical transformations as viewed by prospective teachers. In *Proceedings of the 7th Congress of the European Society for Research in Mathematics Education*, Rzeszów, February 2011.

Xistouri, X. & Pitta-Pantazi, D. (2011). Prospective teachers' cognitive styles and ability in geometry tasks of axial reflection. In G. Vanthournout, L. Coertjens, V. Donche, D. Gijbels, C. Evans, E. Cools, & H. Pedrosa de Jesus (Eds.), *Proceedings of the 16th Annual Conference of the Education, Learning, Styles, Individual Differences Network (ELSIN)* (pp. 433-440). Antwerp, Belgium: Universiteit Antwerpen.

Yanik, H. B. & Flores, A., (2009). Understanding of rigid geometric transformations: Jeff's learning path for translation. *Journal of Mathematical Behavior*, 28(1), 41-57.